

# Approximating the 2-Machine Flow Shop Problem with Exact Delays Taking Two Values

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We study a special case of the 2-Machine Flow Shop problem with exact delays. An instance of the problem consists of  $n$  triples  $(a_j, l_j, b_j)$  of nonnegative integers where  $j$  is a job in the set of jobs  $\{1, \dots, n\}$ . Each job  $j$  must be processed first on machine 1 and then on machine 2,  $a_j$  and  $b_j$  are the lengths of operations on machines 1 and 2, respectively. The operation of job  $j$  on machine 2 must start exactly  $l_j$  time units after the operation on machine 1 has been completed. The goal is to minimize makespan.

The approximability of the general case was studied by Ageev and Kononov in [1]. They proved that the existence of  $(1.5 - \varepsilon)$ -approximation algorithm implies  $P=NP$  and constructed a 3-approximation algorithm. In this paper we consider the case when  $l_j \in \{0, L\}$  for all  $j \in \{1, \dots, n\}$ . In the standard three-field notation scheme this case can be written as  $F2 \mid \text{exact } l_j \in \{0, L\} \mid C_{\max}$ . The problem includes as a special case the classical no-wait 2-Machine Flow Shop problem which is known to be solvable in polynomial time [2]. Our results are the following: we prove that the existence of  $(1.25 - \varepsilon)$ -approximation for  $F2 \mid \text{exact } l_j \in \{0, L\} \mid C_{\max}$  implies  $P=NP$  and present a 2-approximation algorithm.

## References

1. A.A. Ageev, A.V. Kononov, Approximation algorithms for scheduling problems with exact delays. In: Approximation and Online Algorithms: 4th International Workshop (WAOA 2006), Zurich, Switzerland, LNCS, vol.4368 (2007), pp. 1–14.
2. P.C. Gilmore, R.E. Gomory. Sequencing a one-state variable machine: a solvable case of the traveling salesman problem. Operations Research 12 (1964), pp. 655–679.