A Variant of the Cutting-Plane Method without Nested Embedding Sets

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Abstract. We propose a variant of the cutting-plane method [1] with approximating an epigraph of the objective function which is characterized by possibility of updating embedding sets due to dropping cutting planes. Updates occur on the basis of the constructed criterion of approximation quality for the epigraph by polyhedron sets. Convergence of the method is proved and discuss its implementations.

A problem is solved for minimizing convex function $f(x)$ on the convex closed set $D \subset \mathrm{R_n}$. Suppose that x^* is a solution, $f^* = f(x^*)$, $\mathrm{epi}\, f(x) = \{(x,\gamma) \in \mathrm{R_{n+1}} : x \in \mathrm{R_n}, f(x) \le \gamma\}$, $K = \{0, 1, \ldots\}$.

The method is proposed as follows. Choose a point $v \in \mathrm{int\,epi} f(x)$, convex closed sets $M_0 \subseteq \mathrm{R_{n+1}}$, $G_0 \subset \mathrm{R_n}$ such that $\mathrm{epi} f(x) \subset M_0$, $x^* \in G_0$. Select numbers $\bar{\gamma}$, λ_k, τ_k, $k \in K$ according to conditions $\bar{\gamma} \le f^*$, $\lambda_k \in (0,1)$, $\tau_k \ge 0$, $\lambda_k \to 0$, $k \to \infty$, $\tau_k \to 0$, $k \to \infty$. Assign $D_0 = D \bigcap G_0$, $i = 0$, $k = 0$.

1. Find a solution $u_i = (y_i, \gamma_i)$, where $y_i \in \mathrm{R_n}$, $\gamma_i \in \mathrm{R_1}$, of the problem $\min\{\gamma : x \in D_i, (x,\gamma) \in M_i, \gamma \ge \bar{\gamma}\}$. A point $\bar{u}_i \notin \mathrm{int\,epi} f(x)$ is selected in the interval $(v, u_i]$ such that $u_i + q_i(\bar{u}_i - u_i) \in \mathrm{epi} f(x)$ for some $1 \le q_i \le q < +\infty$. If $\bar{u}_i = u_i$, then y_i is a solution of the initial problem. Otherwise, choose a bounded set A_i of generally supported vectors for the set $\mathrm{epi} f(x)$ at the point $\bar{u}_i$.

2. If $\|\bar{u}_i - u_i\| > \lambda_k \|v - u_i\|$, then assign $M_{i+1} = M_i \bigcap \{u \in \mathrm{R_{n+1}} : \langle a, u - \bar{u}_i\rangle \le 0 \ \forall a \in A_i\}$ and go to Step 4. Otherwise, go to Step 3.

3. Choose a point $x_k \in D_i$ such that $f(x_k) \le f(y_i) + \tau_k$, assign $\delta_k = \gamma_i$, $M_{i+1} = M_{r_i} \bigcap \{u \in \mathrm{R_{n+1}} : \langle a, u - \bar{u}_i\rangle \le 0 \ \forall a \in A_i\}$, where $0 \le r_i \le i$, $k := k+1$.

4. Choose a convex closed set $G_{i+1} \subset G_0$ according to condition $x^* \in G_{i+1}$, assign $D_{i+1} = D \bigcap G_{i+1}$, increment i by one, and go to Step 1.

Optimal criterion from Step 1 is proved. It is obtained that sequences $\{x_k\}$, $\{\delta_k\}$ will be constructed together with the sequence $\{u_i\}$.

Theorem 1. *For constructed sequences $\{x_k\}$, $\{\delta_k\}$ it is obtained that* $\lim\limits_{k\to\infty} f(x_k) = \lim\limits_{k\to\infty} \delta_k = f^*$.

References

1. Bulatov, V.P.: Embedding Methods in Optimization Problems. Nauka, Novosib. (1977).