On the approach to set covering with interval uncertainties in weights of sets

Alexander Prolubnikov

Omsk State University, Omsk, Russian Federation
a.v.prolubnikov@mail.ru

The set cover problem with interval uncerainties in weights of sets is under consideration. This problem is naturally arises when we deal with measurements errors or variability of parameters in applied problems. The set cover problem with exact values of weights states as follows. Let $U = \{1, \ldots, m\}$, $S = \{S_1, \ldots, S_n\}$, $S_i \subseteq U$, and let $w : S \to \mathbb{R}_+$ be an additive weight function. A cover of U is a such collection $C = \{S_{i_1}, \ldots, S_{i_k}\}$, $S_{i_j} \in S$, that $U = \cup_{j=1}^{k} S_{i_j}$. We need to find an optimal cover, i.e. the cover with minimal weight $w(C) = \sum_{j=1}^{k} w(S_{i_j})$. To find an optimal cover is *NP*-hard problem for exact values of sets' weights [1]. In the sence of guaranteed accuracy, the greedy algorithm [2] is assimptotically the best polynomial algorithm for obtaining of approximate solution of the problem [3].

Using the greedy algorithm, we build an approach to deal with the set cover problem with interval uncertainties in weights of sets. As a result of implementation of the approach, we have the set of approximate solutions of the problem for all combinations of possible weights. We estimate weights of these solutions. If there is some probability distribution that specified on weights' intervals, we compute probabilities of the solutions. The probability of a solution is a probability that we shall have such a combination of possible exact values of sets' weights that the greedy algorithm will give this solution as an output. As an example of the approach application, we use it to form the set of train's runs and to compute their possible costs.

We show that computational complexity of numerical realization of the approach is non-decreasing step function of intervals' widths. It is shown, that even for small values of m, the interval modification of the set cover problem may be computationally hard to solve. We consider ways of modification of such individual problems that the modified problems are numerically solvable and yet they are the closest problems to the initial ones according to some criteria.

References

1. Garey, M., Johnson, D.: Computers and intractability: a guide to the theory of *NP*-completness. San-Francisco: Freeman (1978).
2. Chvatal, V.: A greedy heuristic for the set-covering problem. Mathematics of operation research. V. 4, N. 3. P. 233–235 (1979).
3. Feige, U.: A threshhold of $\ln n$ for approximating set cover. Journal of the ACM (JACM). V. 45, N. 4. P. 634–652 (1998).