On feedback maximum principle for dynamical systems driven by vector-valued measures

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Abstract. The talk presents a variational necessary optimality condition in the form of the feedback maximum principle due to V.A. Dykhta for a class of terminally constrained dynamical systems of a specific structure, originated in impulsive variational problems with control vector measures and states of bounded variation.

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Given $T,\ y_T > 0,\ c,\ x_0 \in \mathbb{R}^n$, and globally Lipschitz continuous functions $f:\ \mathbb{R}^n \to \mathbb{R}^n$, $G:\ \mathbb{R}^n \to \mathbb{R}^{n\times m}$, we state the optimal control problem (P):

$$I[\sigma] = \langle c, x(T)\rangle \to \min \text{ subject to}$$
$$\dot{x} = (1 - \|v\|)f(x) + G(x)v, \quad x(0) = x_0, \tag{1}$$
$$\dot{y} = 1 - \|v\|, \quad y(0) = 0, \quad y(T) = y_T. \tag{2}$$

A triple $\sigma = (x, y, v)$ is said to be a control process. Let $H = H(x, y; \psi, \xi; v)$ denote the Pontryagian (maximized Hamiltonian) of (P), and (ψ, ξ) the dual of (x, y), being the solution to the adjoint system: $\dot{\psi} = -\frac{\partial}{\partial x}H, \quad \psi(T) = c$, $\xi = \text{const} \in \mathbb{R}$. Introduce the parameterized multivalued map $V_\xi = V_\xi(t, x, y, \psi)$ defined as follows: $V_\xi = \{0\}$, if $y \leqslant t - T + y_T$; V_ξ is the unit sphere in $\mathbb{R}^m$, if $y \geqslant y_T$, and $V_\xi = \operatorname{Arg\,min}\{H|\ \|v\| \le 1\}$, otherwise.

Given a reference process $\bar{\sigma}$, let $\bar{\psi}$ be the respective adjoint state. Denote by $\mathcal{V}_\xi$ the set of single-valued selections w of V_ξ restricted to $\bar{\psi}$, and by $\mathcal{X}(w)$ the set of all solutions – in the senses of Carathéodory and Krasovskii-Subbotin – to system (1), (2) closed-looped by feedbacks w.

Theorem. The optimality of $\bar{\sigma}$ for (P) implies that

$$I[\bar{\sigma}] \le \langle c, x(T)\rangle \ \ \forall\, x \in \mathcal{X}(w),\ w \in \mathcal{V}_\xi,\ \xi \in \mathbb{R}.$$

In the talk, we discuss some consequences of the theorem, and its application to numerical implementation of impulsive control problems.

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