Impulsive Behavior and Hybrid Properties of Control Mechanical Systems

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Abstract. The talk presents some recent results in the area of modeling and optimal control for a class of specifically mixed-constrained impulsive dynamical systems driven by Borel measures.

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We address control dynamical systems described by measure differential equations of the form

$$dx = f(x)dt + G(x)d\mu.$$

Impulsive effects are regarded to be performed by vector-valued Borel measure μ, and states are functions of bounded variation.

The interface between states and the control measures is presented by mixed conditions of a complementarity nature being constraints on one-sided limits of a state before and after its jumps:

$$x(t^-) \in \mathcal{Z}_- \text{ and } x(t) \in \mathcal{Z}_+ \quad |\mu|\text{-almost everywhere,}$$

where $|\mu|$ is the total variation of μ.

A practical motivation emerges from Lagrangian mechanics and contact dynamics as a modeling challenge for such effect as impulsive forces of unilateral contact of rigid bodies, or impactively blockable degrees of freedom.

Our goals are: 1) to invent a constructive description of the closure of the trajectory tube for the stated model; 2) design an approximation of the mixed-constrained measure-driven system by ordinary control processes driven by measurable bounded controls, and 3) establish an equivalent transformation of a related optimal control problem to an ordinary variational model.

In conclusion of the talk, we discuss an application of the obtained theoretical results to numerical simulation of hybrid systems.

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