Estimates of Transients in Discrete Time Linear Systems

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Analysis of transients in dynamical systems is of great importance, e.g., see [1]. However, typically, it is assumed that the initial conditions are zero, and the system is subjected to exogenous input signals. Not much work has been performed for *input-free* systems with *nonzero* initial conditions; e.g. see [2] for a recent publication. We say that a stable system experiences peak if its solution deviates from the unit-norm initial conditions at finite time instants before converging to zero.

The situation with discrete-time systems is different. Though the stability theory for difference equations is well-developed, see [3], to the best of our knowledge, no attention has been paid to the analysis of similar peak effects.

In this paper we consider discrete-time linear systems in the form $x_{k+1} = Ax_k,\ x_k \in \mathbb{R}^n$, and discuss several results in this direction. First, for Schur stable systems we provide upper bounds on deviations of trajectories with arbitrary initial conditions of unit Euclidean norm; equivalently, we estimate the peak of $\|A^k\|$. Second, admitting for the presence of a control input, we design a linear feedback that minimizes the peak of the closed-loop system. These two estimates are obtained via use of linear matrix inequalities and solutions of semidefinite programs.

Next, we consider systems with companion-form Schur stable matrices (equivalently, stable scalar difference equations of nth order) with equal real eigenvalues $\lambda_i \equiv \rho$, $\rho \in (0,\ 1)$, and initial conditions $(0, 0, \dots, 1)$ and obtain the exact values of the magnitude of peak and peak instant. We show that, as the order of the system grows and/or the value of ρ increases, the magnitudes of both peak and peak instant grow. The results of numerical experiments are also presented.

References

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