On a Polynomial Algorithm for the Resource Constrained Multi-Project Scheduling Problem on Random Instances

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We consider a particular case of Resource Constrained Project Scheduling Problem (RCPSP) with single resource of R units and precedence constraints set by oriented acyclic graph G consisting of m connectivity components (that play the role of several independent sub-projects of a certain big project).

An approach of solving this problem is based on two ideas. First, one can note that any feasible solutions of the strip packing problem can serve as feasible solutions of RCPSP with single resource in case of absence of any precedence relations. Thus, approximation algorithms for strip packing problem can be applied to this particular case of RCPSP.

Second, due to acyclic nature of each sub-project component, we can enumerate jobs in each component in the way agreed with the precedence relations; i.e., a for any precedence relation, a predecessor has less number then the successor. Then, we can schedule the set of jobs with number 1 in each project ("layer 1 jobs"), then the set of jobs with number 2 ("layer 2"), using the completion time of previous set a the starting time of current set, and continue in the similar way for all the other layers.

Random instances are generated the following way: each of n jobs is independently getting duration and resource consumption according to some discrete distribution given by the $R \times L$ matrix (p_{rl}). Then, each job is assigned to some of m sub-projects and acyclic set of edges is added to each component.

Using based on results from [1] we prove the following theorem.

Theorem 1. *Assume that each subproject contains k jobs, i.e., $n = mk$. Then, in cases*

- *when (p_{lr}) is L-asymmetric and $k^2 W = o(\frac{m}{\ln} m)$*
- *when (p_{lr}) is L-regular and $\frac{k^2 W}{\beta_p} = o(\frac{m}{\ln} m)$, where $\beta_p = \frac{1}{WH} \sum_{r=1}^{R} \sum_{l=1}^{L} rlp_{rl}$*

the multi-project RCPSP with single resource is solved asymptotically optimally in polynomial time.

References

1. Gimadi, E., Zalyubovskii, V., Sharygin P.: The problem of strip packing: An asymptotically exact approach. Izv. Vyssh. Uchebn. Zaved. Mat., 12, 3444 (1997)

* This work was supported by grants 16-31-00389,16-07-0168 and 15-01-00976 from the Russian Foundation for Basic Research.