On minimizing supermodular set functions on matroids

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Let I be a finite set. A *matroid* on I is a pair $M = (I, \mathcal{A})$, where $\mathcal{A} \subseteq 2^I$ is a family of subsets of I satisfying the following two axioms:
($A1$) $(A \in \mathcal{A},\ A' \subseteq A) \Rightarrow A' \in \mathcal{A}$;
($A2$) $(A, A' \in \mathcal{A},\ |A| = |A'| + 1) \Rightarrow \exists\ a \in A \setminus A'\ : A' \cup \{a\} \in \mathcal{A}$.
The sets $A \in \mathcal{A}$ are called *independent sets* of M.

A set function $f : 2^I \to \mathbf{R}_+$ is called *supermodular*, if for all $A, B \subseteq I$

$$f(A \cup B) + f(A \cap B) \geq f(A) + f(B).$$

We consider the combinatorial optimization problem:

$$\min\{f(X) : X \in \mathcal{B}\}, \tag{1}$$

where $f : 2^I \to \mathbf{R}_+$ is a nondecreasing supermodular set function, $f(\emptyset) = 0$, and $\mathcal{B}$ is the family of all maximal independent sets (bases) of a matroid $M = (I, \mathcal{A})$.

The well-known NP-hard minimization p-median problem [1] can be reduced to this problem.

We present a performance guarantee of the approximation greedy algorithm for problem (1) using the notion of curvature of the objective function $f(X)$. As a corollary we obtain a bound on worst-case behaviour of the greedy algorithm for the general minimization p-median problem that improves and complements the known bounds [2, 3].

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References

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