Finding Bounded-below-Diameter Minimum-edge disjoint Spanning Trees

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The classic minimum spanning trees problem is to find k edge-disjoint spanning trees in given undirected weighted graph. It can be solved in polynomial time [1]. In the diameter bounded below minimum spanning trees problem there is an additional requirement: a diameter of every spanning tree must be not less than some predefined value d. The diameter of a tree is the number of edges on the longest path between two leaves in the tree. We will abbreviate this problem as k-MSTBB. The k-MSTBB is NP-hard since for $k = 1$ and $d = n - 1$, where n is the number of vertices in the graph, it is equivalent to the Hamiltonian path problem. The k-MSTBB for $k = 1$ was introduced in [2] and the asymptotically optimal algorithm for the case of uniform distribution on a segment was presented. k-MSTBB for arbitrary k and uniform distribution was studied in [3].

In current work we consider the k-MSTBB on compete n-vertices undirected graphs where edges weights are independent identically distributed random variables with discrete distribution on a set $\{1, \dots, b_n\}$. We propose a polynomial time algorithm to solve the problem.

The algorithm builds k edge-disjoint Hamiltonian chains with d edges each on the same $d + 1$ vertices. Then by the algorithm by Roskind and Tarjan [1] algorithm finds k edge-disjoint spanning trees containing these chains.

A probabilistic analysis was performed under conditions that graph edges weights are identically independent distributed random variables with uniform distribution on a set $\{1, b_n\}$, $0 < b_n < \infty$. Also it is supposed that $d = d_n$ goes to infinity as n goes to infinity. It was shown that proposed algorithm finds optimal solution with high probability (whp), i.e. probability tends to 1 as n goes to ∞.

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References

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