Interval regularization for systems of linear algebraic equations

Sergey P. Shary

Institute of Computational Technologies SB RAS, Novosibirsk, Russia,
shary@ict.nsc.ru,
WWW home page: http://www.nsc.ru/interval/shary

The present work is devoted to the solution of ill-conditioned systems of linear algebraic equations. We propose a new regularization method based on ideas and methods of interval analysis.

Solving systems of linear algebraic equations of the form

$$Ax = b,$$

with a matrix A and right-hand side vector b, is one of the most important components of many modern computational technologies. However, if the matrix A is ill-conditioned (almost singular), then finding a reliable solution to such a system presents considerable difficulties.

To improve the stability of the solution, we "intervalize" the system $Ax = b$, i.e., we replace it by an interval system $\boldsymbol{A}x = \boldsymbol{b}$ of linear algebraic equations with a matrix $\boldsymbol{A}$ whose elements are obtained by blowing up the elements from A, and the right-hand side $\boldsymbol{b}$ is obtained from the vector b by the similar procedure. As a result, the "inflated" matrix of the system acquires close well-conditioned matrices, for which the solution of the corresponding systems is more stable.

As a pseudo-solution of the original system of linear algebraic equations, a point is taken from the *tolerable solution set* to the intervalized linear system $\boldsymbol{A}x = \boldsymbol{b}$. The rationale is that the tolerable solution set (see [1, 2]), defined as

$$\Xi_{tol} = \big\{\, x \mid (\forall A \in \boldsymbol{A})(\exists b \in \boldsymbol{b})(Ax = b) \,\big\},$$

is the intersection of partial solution sets corresponding to separate point matrices A from $\boldsymbol{A}$, thus being the most stable among the solution sets to the interval linear system $\boldsymbol{A}x = \boldsymbol{b}$.

To find a point from Ξ_{tol}, one can apply either the subdifferential Newton method [2] or the technique based on the use of the so-called recognizing functional [1, 2]. The latter option leads to a non-smooth convex optimization problem, for which efficient numerical methods have been elaborated recently.

References

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