Scheduling Problems with Controllable Processing Time to minimize Minmax and Quadratic Objectives

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In scheduling with controllable processing times (SCPT) the actual durations of the jobs are not fixed in advance, but have to be chosen from a given interval. For a SCPT model, two types of decisions are required: (i) each job has to be assigned its actual processing time, and (ii) a schedule has to be found that provides a required level of quality.

The jobs of set $N = \{1, 2, \ldots, n\}$ have to be processed either on a single machine M_1 or on parallel machines $M_1, M_2, \ldots, M_m$, where $m \geq 2$. For each job $j \in N$, its processing time $p(j)$ is not given in advance but has to be chosen from a given interval $[l(j), u(j)]$. That selection process is often seen as *compressing* the longest processing time $u(j)$ down to $p(j)$. The value $x(j) = u(j) - p(j)$ is called the *compression amount* of job j.

Each job $j \in N$ is given a *release date* $r(j)$, before which it is not available, and a *deadline* $d(j)$, by which its processing must be completed. In the processing of any job, *preemption* is allowed, so that the processing can be interrupted on any machine at any time and resumed later, possibly on another machine. It is not allowed to process a job on more than one machine at a time, and a machine processes at most one job at a time. A schedule is called *feasible* if the processing of a job $j \in N$ takes place in the time interval $[r(j), d(j)]$.

Each job can be associated with several weights $w_T(j), w_M(j)$ and $w_Q(j)$, which serve as coefficients that define an objective function that depend on the compression amounts. Typical functions include (i) total compression cost $\Phi_\Sigma = \sum_{j \in N} w_T(j)\, x(j)$, (ii) maximum compression cost $\Phi_{\max} = \max\{x(j)/w_M(j) \mid j \in N\}$, and (iii) quadratic compression cost $\Phi_Q = \sum_{j \in N} \left(w_Q(j)\, x(j)^2 + w_T(j)\, x(j)\right)$.

Problems that involve minimization of function Φ_Σ are well-studied within the SCPT area. The function $\Phi_{\max}$ is a popular objective within the body of research on scheduling with imprecise computation. The quadratic function Φ_Q, although obviously relevant, has not received considerable attention so far.

In this paper, we report on the progress in solving the problems that involve minimization of these functions, including a multicriteria environment, either for the lexicographically ordered criteria or in the Pareto sense. For many problems in this range polynomial-time algorithms can be designed.