The Regularization Parameter Choice in the Nonclassical Variational Problems Statements of the Inverse Problems

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The solution of the inverse problems in many cases can be reduced (after the appropriate sampling) to the solution of system of the linear algebraic equations (SLAE) which matrix has a ill conditionality (and it is possible also degeneracy). In these cases for obtaining the steady solution to use a Tikhonov regularization method with classical quadratic functional. The resulting regularizing algorithm is linear (in case of the given parameter of regularization). Unfortunately, the property of linearity does not allow us to find well a solution vector, in which there are abrupt changes in the amplitude of the projections in combination with the intervals at which the projections are almost constant. Such solutions can be called contrast solutions. To overcome this difficulty in this paper, the regularized solution ϕ_α of SLAE is found from the condition of a minimum of the functional:

$$F_\alpha(\phi) = ||\tilde{f} - K\phi||_2^2 + \alpha||\phi||_1.$$

The presence of a functional $||\phi||_1 = \sum_i |\phi_i|$ determines the discontinuous character of the gradient, that generates the known difficulties connected to convergence of iterative procedures. In the literature, an algorithm for minimizing the functional using wavelet functions is proposed. However, the choice of the regularization parameter itself, which has a significant effect on the accuracy of the regularized solutions obtained, has not been solved.

Therefore, in this paper the main attention is paid to the construction of a statistical algorithm for estimating the optimal regularization parameter that minimizes the mean square error of the regularized solution and based on checking the statistical hypotheses about the residual vector. The carried out researches have shown that the proposed algorithm for choosing the regularization parameter allows to estimate with an acceptable accuracy the optimal value of the regularization parameter and can be successfully used for solving practical invers problems.

Keywords: ill conditioned SLAE, variational problems with non-quadratic functional, estimation of the optimal regularization parameter.