# Modified duality method for solving model crack problem

Ellina Vikhtenko and Marina Tchervyakova

Pacific National University,
Tikhookeanskaya 136, 680035 Khabarovsk, Russia
vikht.el@gmail.com,m.tchervyakova@yandex.ru

**Abstract.** We consider the duality method based on the use of modified Lagrangian functional for solving a model of elastic problem with a crack. We construct Uzawa method for finding a saddle point, proved the convergence theorems. The results of numerical experiments are given.

**Keywords:** model problem with a crack, variational inequality, modified Lagrangian functional, Uzawa method

This paper is devoted to the analysis of model, a wide interest in which is manifested in the recent years. It is a model problem of elastic deformation of the body containing a crack. The formulation of this type of problem can be found in monograph by A M Khludnev [1]. Model proposed here differs from classical approach to the crack problem because it is characterized by the nonlinear boundary conditions on crack faces. Suitable boundary conditions are written as inequalities which provide mutual nonpenetration between crack faces. From the standpoint of mechanics such models are more preferable than the linear classical models.

The modified Lagrangian functional for the first time were developed and investigated for solving the problem of finite-dimensional optimization. Their emergence was related to the fact that classical Lagrangian functionals that are linear functions of the dual variables are not suitable for solving the singular optimization problems. The construction of modified Lagrangian function (functional) actually comprises regularization of dual variables. In last time the Lagrangian multiplier method is successfully applied to the solution of infinite-dimensional variational inequalities in mechanics. Applying a similar scheme for solving the crack problem complicated by the fact that in the neighborhood of the crack faces the regularity of the solution may be arbitrary bad, and the dual problem may be unsoluble. Despite this problem, it is possible to justify the duality scheme to solve the crack problem, as well as equality of duality for the original and dual problems.

## References

1. Khludnev, A. M.: Elasticity Problems in Non-smooth Domains. Fizmatlit, Moscow (2010)