Minimizing a sensitivity function as boundary-value problem of terminal control

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Abstract. We consider a finite-dimensional optimization problem of minimizing a sensitivity function under constraints as a boundary-value problem in terminal control problem. A saddle-point method for solving problem is proposed. The method's convergence to solution of the problem in all the variables is proved.

Let us consider a problem of minimizing a sensitivity function on set Y of admissible values of y:

$$\varphi(y) = f(x^*) = \mathrm{Min}\{f(x) \mid g(x) \leq y,\ x \in X \subset \mathrm{R}^n\}, \tag{1}$$

$$y^* \in \mathrm{Argmin}\{\varphi(y) \mid y \in Y \subset \mathrm{R}^m_+\}, \tag{2}$$

where $f(x)$ is a scalar convex function; $Y = \{y \geq 0 \mid G(y) \leq d,\ d \in \mathrm{R}^m_+\}$; $g(x)$, $G(y)$ are vector functions with convex components; d is a fixed vector; X, Y are convex closed sets. Solutions to (1) and (2) form convex closed sets X^*, Y^* with $x^* \in X^*$, $y^* \in Y^*$. Problem (1),(2) plays a role of a boundary-value problem in a dynamic problem of terminal control.

To solve (1),(2) we use a saddle-point approach based on reducing the problem to finding a saddle point of Lagrange function $L(x, y^*, p) = f(x) + \langle p, g(x) - y^*\rangle$. Dual extraproximal iterative method has been used to implement this approach:

$$\bar{p}^k = \pi_+(p^k + \alpha(g(x^k) - y^k)), \tag{3}$$

$$y^{k+1} = \pi_Y(y^k + \alpha\bar{p}^k), \tag{4}$$

$$x^{k+1} = \mathrm{argmin}\left\{\frac{1}{2}|x - x^k|^2 + \alpha(f(x) + \langle\bar{p}^k, g(x) - y^k\rangle) \mid x \in X\right\}, \tag{5}$$

$$p^{k+1} = \pi_+(p^k + \alpha(g(x^{k+1}) - y^{k+1})). \tag{6}$$

Theorem 1 (On convergence of method). *If solution to* (1), (2) *exists then sequence* (p^k, x^k, y^k) *of dual extraproximal method* (3)-(6) *with parameter* α *satisfying the condition* $0 < \alpha < \min\{1/(2|g|), 1/2\}$ *converges monotonically in norm to one of solutions of the problem* (p^*, x^*, y^*) *as* $k \to \infty$ *for all* (p^0, x^0, y^0).

References

[1] Antipin, A.S.: Sensibility function as convolution of system of optimization problems. Optimization and Optimal Control. Chinchuluun A., Pardalos P.M., Enkhbat R., Tseveendorj I. (Eds.). Springer, 1–22 (2010)