Efficient optimal algorithm for the Quasipyramidal GTSP*

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The Traveling Salesman Problem (TSP) is one of the famous NP-hard combinatorial optimization problems. Although TSP is intractable and hardly approximable in its general case, there are restricted settings of the problem, which can be solved to optimality in polynomial time. For instance, it is known [1] that, for any non-negative weight function w, in weighted complete (di)graph $G = (\mathbb{N}_n, E, w)$, an optimal *pyramidal* Hamiltonian cycle, i.e. the closed tour

$$1, i_1, \ldots, i_{r-1}, i_r = n = j_{n-r}, j_{n-r-1}, \ldots, j_1, 1, \tag{1}$$

for which $i_s < i_{s+1}$ and $j_t < j_{t+1}$ for each $0 \leq s < r-1$ and $0 \leq t < n-r-1$, can be found in time of $O(n^2)$.

In [2, 3] this result is extended to the special cases of quasipyramidal tours (called by the authors as tours with *step-backs* and *jump-backs*). Actually, for some fixed l, tour (1) is called *l-quasipyramidal* if $i_p \leq i_q + l$ and $j_u \leq j_v + l$ for any $1 \leq p < q \leq r$ and $1 \leq u < v \leq n-r$. As it is proven in [3], an optimal l-quasipyramidal tour can be found in time of $O(8^l n^2)$.

In this paper we propose an extension of the aforementioned results to the case of the Generalized Traveling Salesman Problem, where a partial order on the vertex set V of a given graph $G = (V, E, w)$ is induced by the linear ordering of the clusters $V_1, \ldots, V_k$. Along with presenting the appropriate polynomial time algorithms finding optimal pyramidal and quasipyramidal tours in this case, we propose novel extensions of the well-known Demidenko and van der Veen sufficient conditions providing existence of an optimal GTSP solution in the subclass of (quasi)pyramidal tours.

References

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