On $(1,l)$-coloring of incidentors of some classes of graphs

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An *incidentor* in a directed loopless multigraph $G = (V, E)$ is an ordered pair (v, e) where $v \in V, e \in E$ and the arc e is incident with the vertex v. It is convenient to treat the incidentor (v, e) as a half of the arc e adjoining to the vertex v. Each arc $e = uv$ has two incidentors: the *initial* one (u, e) and the final one (v, e). Two incidentors adjoining the same vertex are called *adjacent*. An incidentor coloring is an arbtrary function $f : I \longrightarrow Z_+$, where Z_+ is the set of positive integers (colors). The incidentor coloring is called (k, l)-coloring if: 1) all adjacent incidentors have different colors; and 2) for every arc, the difference between the colors of its final and initial incidentors is in $[k, l]$. The minimum number of colors for which such coloring is possible is denoted by $\chi_{k,l}(G)$.

The notion of incidentor (k, l)-coloring was introduced in [1]. Some bounds on $\chi_{k,l}(G)$ were proved in [2–4]. In particular, it was proved in [2] that for every graph G of maximum degree Δ and $l \geq \lceil \Delta/2 \rceil$ the bound $\chi_{k,l}(G) \leq \Delta + k$ holds. In this paper we prove the same bound for $k = 1$ and $l = \lceil \Delta/2 \rceil - 1$.

The $(1, 1)$-coloring of incidentors is particulary interesting since the only series of graphs G having $\chi_{k,l}(G) > \Delta + k$ was constructed in [2] for $k = l = 1$ and odd Δ. Moreover, all these graphs had no perfect matching. The author conjecture that evrey graph G with a perfect matching satisfies the bound $\chi_{1,1}(G) \leq \Delta + 1$ and prove this fact for the class of prisms.

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