An Approximation Algorithm for the Euclidean Maximum Connected k-Factor

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Given an n-vertex undirected complete graph $G = (V, E)$ without self loops, a weight function $w : E \to \mathbb{R}_+$ and a positive integer $k \geq 2$, the problem is to find a spanning connected k-regular subgraph (connected k-factor) in G of maximum or minimum total weight. This problem is closely related to the network design, where connectivity and degree requirements are common. It is known that the problem is polynomially solvable if there is no requirement for the subgraph to be connected, and NP-hard otherwise. Note that the connected 2-factor problem is the Traveling Salesman Problem.

For the minimum metric connected k-factor problem there is a polynomial approximation algorithm with constant approximation ratio [2].

As long as the connected k-factor problem is a natural generalization of the TSP, and the maximum TSP is in some ways easier than the minimum TSP, the case of the maximum connected k-factor is also of interest.

Paper [1] provides an approximation algorithm for the maximum connected k-factor problem with the relative error $\varepsilon = O\left(1/k^2\right)$ and with $O(kn^3)$ running-time. We note that with more accurate calculations the relative error can be improved to $\varepsilon = O\left(1/k^3\right)$. It is clear that this algorithm is asymptotically optimal for large k that tends to infinity as n grows.

Here we concentrate on the Euclidean maximum connected k-factor problem where the weight of an edge is the Euclidean distance between its endpoints. For this problem in the case of constant or small $k = o(n)$ we present an asymptotically optimal approximation algorithm, which runs in $O(n^3)$ time.

References

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