Convex optimization in Hilbert space with applications to ill-posed problems

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1 Abstract

We propose rather general approaches (based on the gradient type methods) to solve convex optimization problems in Hilbert space [2]. We are interested in the case when Hilbert space has infinite dimension. It doesnt typically allow to calculate the exact value of the gradient (Frechet derivative). So that we have a tradeoff between the cost of one iteration and the number of required iterations: one can calculate gradient exactly, so the convergence is fast (we may use fast gradient descent), but the cost of one iteration is large, vice versa, one can calculate the gradient roughly so the convergence is slow (we can use only robust the simple gradient method), but the cost of the one iteration is cheap. This investigation is motivated by the class of ill-posed problems for elliptic initial-boundary value problems (the Cauchy problem for the Laplace equation [1]).

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References

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