On the skeleton of the pyramidal tours polytope*

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The skeleton of the polytope P is the graph whose vertex set is the vertex set of P and edge set is the set of 1-faces of P. There are two results on the traveling salesman polytope $\mathrm{TSP}(n)$ of interest to us: the question whether two vertices of the $\mathrm{TSP}(n)$ are nonadjacent is NP-complete [4], and the clique number of the $\mathrm{TSP}(n)$ skeleton is superpolynomial in dimension [1]. It is known that this value characterizes the time complexity in a broad class of algorithms based on linear comparisons [2].

Hamiltonian tour is called a pyramidal if the salesman starts in city 1, then visits some cities in increasing order, reaches city n and returns to city 1 visiting the remaining cities in decreasing order. Pyramidal tours have two nice properties. First, a minimum cost pyramidal tour can be determined in $O(n^2)$ time by dynamic programming. Second, there exist certain combinatorial structures of distance matrices that guarantee the existence of a shortest tour that is pyramidal [3].

We consider the skeleton of the pyramidal tours polytope $\mathrm{PYR}(n)$ that is defined as the convex hull of characteristic vectors of all pyramidal tours in the complete graph K_n. We describe necessary and sufficient condition for the adjacency of the $\mathrm{PYR}(n)$ polytope vertices. Based on that, we establish following properties of the $\mathrm{PYR}(n)$ skeleton.

Theorem 1. *The question whether two vertices of the* $\mathrm{PYR}(n)$ *are adjacent can be verified in linear time* $O(n)$.

Theorem 2. *The clique number of the* $\mathrm{PYR}(n)$ *skeleton is* $\Theta(n^2)$.

Thus, the clique number correlates with the time complexity $O(n^2)$ of dynamic programming for pyramidal traveling salesman problem.

References

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