Approximation algorithms for intersecting straight line segments with equal disks*

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In this work approximation algorithms for the following problem are given. Intersecting Plane Graph with Disks (IPGD): *given a simple plane graph* $G = (V, E)$ *and a constant* $r > 0$, *find the smallest cardinality set* $C \subset \mathbb{Q}^2$ *of points (disk centers) such that each edge* $e \in E$ *(which is a straight line segment) is within (Euclidean) distance* r *from some point* $c = c(e) \in C$ *or, equivalently, the disk of radius* r *centered at* c *intersects* e.
It can be reduced to the classical geometric Hitting Set problem with the set of objects $\{\{x \in \mathbb{R}^2 : d(x, e) \leq r\} : e \in E\}$ and the ground point set equal to $\mathbb{R}^2$, where $d(x, e)$ denotes Euclidean distance between a point $x \in \mathbb{R}^2$ and a segment $e \in E$. When segments of E have zero lengths, IPGD coincides with the known Disk Cover problem. Designing algorithms for it finds its applications in network security analysis [1]. The IPGD problem is intractable even in its simple cases.

Theorem 1. IPGD *is NP-complete even if segments from* E *do not intersect.*

Analogous hardness results are proved for special graphs G [2] of practical interest. Using approaches of [3] and [4], we give

Theorem 2. *A* $400e$*-approximate algorithm exists for* IPGD *which can be implemented in* $O(|E|^4 \log^2 |E|)$ *time and* $O(|E|^2)$ *space.*

We improve on the approach of [3] applied directly to IPGD both in complexity and approximation factor by using new ideas and data structures. Sharper approximation algorithms are also given for special graphs G of practical interest.

References

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