Minimization of polyhedral function over hypercube using projections

Maxim Demenkov

Institute of Control Sciences, Profsoyuznaya 65, 117997 Moscow, Russia
max.demenkov@gmail.com

We consider a rather classical problem of nonsmooth optimization — minimization of polyhedral function with interval constraints on variables, i.e.

$$\min_{\mathbf{B}} \max_{i=1,\ldots,N} a_i^T x + c_i, \;\; \mathbf{B} = \{x \in \mathbb{R}^n : x_{imin} \leq x_i \leq x_{imax}, i = 1, ..., n\}.$$

It is clear that one can apply linear programming or some kind of subgradient algorithm to solve this problem. Despite that, we investigate a new approach based on conversion of the problem into new setting — finding an intersection between a special polytope (*zonotope*) and a line [1].

Zonotope is an affine transformation of m - dimensional cube:

$$\mathbf{Z} = \{z \in \mathbb{R}^n : z = z_0 + Hw, \; ||w||_\infty \leq 1\}, \; w \in \mathbb{R}^m, \; n \leq m.$$

First, we transform our problem into the form

$$\gamma \to \min, \;\; Ax + c + y = \overline{1}\gamma,$$

where $\overline{1}$ is a vector of ones and y is a vector of auxiliary variables (also box-constrained). Then our problem is reduced to the following:

$$\gamma \to \min, \;\; \overline{1}\gamma \in \mathbf{Z}, \;\; w = [x^T \; y^T]^T.$$

If we know an interior point of $\mathbf{Z}$ on the line $\overline{1}\gamma$, it is possible to derive a linearly convergent algorithm based on bisection of interval on the line. At each iteration (the number of iterations can be computed in advance for the given accuracy) we apply an algorithm (e.g. Frank-Wolfe [2] or Nesterov fast gradient) to find a projection of a point on the line onto the zonotope (which is equivalent to projection onto a hypercube). Then (if the current point is outside $\mathbf{Z}$) we take a point of intersection between the line and the achieved level set of distance function as our next point. The algorithm can be used as e.g. a part of d.c. optimization in the style of [3].

References

1. Fujishige, S., Hayashi, T., Yamashita, K., Zimmermann, U.: Zonotopes and the LP-Newton Method. Optimization and engineering, 10, 193-205 (2009).
2. Lacoste-Julien, S., Jaggi, M., On the Global Linear Convergence of Frank-Wolfe Optimization Variants. In: NIPS (2015).
3. Strekalovsky, A.S., Gruzdeva, T.V., Orlov A.V.: On the Problem of Polyhedral Separability: a Numerical Solution. Automation and Remote Control, 76(10), 1803-1816 (2015).