# Faster approach for exhaustive search over graphs with certain properties

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**Abstract.** We present new techiques for exhaustive search over graphs with certain properties. We apply them to the cubic version of the Erdős-Gyárfás conjecture and computationally show that the smallest cubic graphs without 4,8,16 cycles have 78 vertices.

**Keywords:** Erdős-Gyárfás conjecture; cubic graphs; exhaustive search

The cubic version of the Erdős-Gyárfás conjecture asserts that every cubic graph (regular graphs of degree 3) contains a cycle whose length is a power of 2.

Using computer search Royle and Markström showed that any example must have at least 17 vertices, and a cubic example must have at least 30 vertices [Ma1]. They also found 4 minimal cubic graphs of order 24 without 4,8 cycles (but having cycle of length 16). Exoo constructed cubic graph of order 78 without 4,8,16 cycles [Ex1] and asked whether there are smaller examples here.

We show that the graph found by Exoo is indeed the smallest example of a cubic graph without 4,8,16 cycles, and also present 5 other such graphs.

Our approach is based on a framework developed by McKay for isomorphism-free exhaustive generation of combinatorial objects [Mc1] and fast generation of cubic graphs by Brinkmann [Br1]. We implemented a program similar to the program `minibaum` written by Brinkmann with a few special modifications.

The main idea of our approach is using a probabilistic checking of every node of the search tree. This method increases the number of nodes we have to consider, but significantly reduces the average time for checking every particular node. Thus, it reduces the total computing time. Our program turned out about 300 times faster than the `minibaum` program.

Our techniques can be applied to other graph problems too.

## References

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