Norm variability in Newton method for underdetermined systems of equations

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Newton method may serve as a tool for solution of underdetermined systems of algebraic (differentiable) equations $P(x) = 0,\ P : \mathbb{R}^n \to \mathbb{R}^m,\ m < n$. It is usually written via pseudo-inverse matrix, which correspond to Euclidean norms in pre-image and image spaces [1, 4]:

$$x^{k+1} = x^k - \alpha(P'(x^k))^\dagger P(x^k),$$

The same method can be used to explore image set of a non-linear differentiable mapping $\{g(x) : x \in \mathbb{R}^n\} \subseteq \mathbb{R}^m$, resulting in equations of type $g(x) = \gamma y$, with chosen direction y.

We propose variable-norm setup for Newton method as

$$z_k = \arg \min_{P'(x^k)z = P(x^k)} \|z\|,$$
$$x^{k+1} = x^k - \alpha z^k,$$

Using generic convergence conditions, based on technique of [2, 3] we study different norm combinations, choice of norms for image exploration problems, as well as constant estimation issues related with norm choice.

References

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