Semidefinite relaxations for the optimal power flow: robust or fragile?

Elena Gryazina

Institute for Control Sciences RAS and SkolTech Center for Energy Systems, Moscow, Russia gryazina@gmail.com

The optimal power flow problem (OPF) aims at constructing optimal operating regime of a power system that minimizes generation cost or network losses subject to engineering and physical network constraints. The OPF problem is a part of System Operator's daily routine, however most of the industrial solvers use the linearized (so-called DC) formulation. Original quadratic formulation (AC OPF) is more precise, but harder to solve. Among different optimization approaches we investigate semidefinite relaxations [1], [2] that demonstrated their efficiency for the number of industrial benchmarks. However, for transmission networks there is no theoretical validation of the conditions that guarantee the exactness of relaxation. Motivated by the simple illustrative example [3], we investigate the situations when the relaxed formulation is unable to retrieve feasible solution.

We demonstrate that slight modifications to the objective function and/or constraints can lead to the failure of the relaxation. Particularly, two modified problems were formulated: in the first problem constraints on reactive power were omitted, and in the second one, several buses of the network were allowed to consume unlimited amount of power, but not to generate. For simplicity, we assumed the first bus to be the only generator in the grid and minimized its active power injection under constraints on demand on power and voltages at each bus. These two problems were tested on IEEE14 network. In both cases the rank constraint was violated that marks the failure of semidefinite relaxation.

For the moment semidefinite relaxations for OPF are on the way to get embedded in the industrial software. Numerous papers report zero gap between initial and relaxed problems. Our research provides a step towards justification the optimality of the relaxed solution as well as marks the warnings that should be taken into account.

References

1. J. Lavaei, and S.H. Low, "Zero duality gap in optimal power flow problem," IEEE Transactions on Power Systems, vol. 27, no. 1, pp. 92- 107, 2012.
2. S. Bose, et al., "Equivalent relaxations of optimal power flow," IEEE Transactions on Automatic Control, Vol. 60. - No. 3. - P. 729-742, 2015.
3. Molzahn D. K., Baghsorkhi S. S., Hiskens I. A. Semidefinite relaxations of equivalent optimal power flow problems: An illustrative example //Circuits and Systems (ISCAS), 2015 IEEE International Symposium on. IEEE, 2015. P. 1887-1890.