Optimal control for discrete-time stochastic systems w.r.t. the probabilistic performance index

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Abstract. The optimal control problem for discrete-time stochastic systems with probabilistic performance index is considered. New results of qualitative research based on the dynamic programming are presented.

Keywords: the discrete-time stochastic systems, the optimal control, the probabilistic performance index, the dynamic programming

The problem in question of this report is stochastic optimal control of discrete-time system w.r.t. the probabilistic performance index. Such models arise in the aerospace, economics and robotics. The existing numerical methods for solving such problems are ineffective because of the known curse of dimension.

The probabilistic performance index is defined as probability that a certain precision functional does not exceed a certain admissible level. Here the precision functional itself characterizes the accuracy of the control system but depends on the trajectory of the stochastic system. One example of such a precision functional is the terminal miss of a guidance system.

In this report we present new results concerning of properties of the Bellman function on the basis of utilisation of the boundedness of the probability.

Using the dynamic programming and the properties of the Bellman function we find two-sided bounds on the Bellman function under general assumptions about the control system, the domain of feasible controls, the precision functional and the random noise distribution. It is proved that under certain conditions the solution of the original control problem coincides with one of the stochastic programming problem of a certain structure.

As an example, the optimal control problem of a portfolio of securities with one risk-free and a given number of risk assets is considered. Using a two-sided estimate of the Bellman function, we prove the asymptotic optimality of the risk strategy.

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