Binary Cuts Algorithm for Mixed Integer Programming Problems

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A common feature shared by most implementations of mixed integer linear programming (milp) problems is the NP class membership, large dimensions, and complexity of constraint structures. The focus of this paper is on a method of solving milp problems, which is based on binary cuts (BCs) [1]. One of its algorithms, a hybrid algorithm based on binary cuts and branches (binary cut-and branch algorithm (BCBA)), which combines the idea of the branch-and-bound method with the construction of cutting planes, is extended to milp problems. The results of this extension are discussed. The following problem is considered:

$$\gamma(x) = {c^1}^T x + {c^2}^T y + const \rightarrow \max \tag{1}$$

$$A^1 x + A^2 y \le b, \bar{0} \le x \le \bar{1}, y \ge \bar{\bar{0}}, \tag{2}$$

$$x \in I_2^{n^1}, \tag{3}$$

which is a milp problem with Boolean variables and continuous variables. Suppose x^0, y^0 is the solution of the relaxed problem (1)-(2); $[\cdot]$ is the integer part of number; and $\beta_0 = \widehat{\alpha}^T x^0$, where $\widehat{\alpha}_j \in \{0,1\}$, $j = \overline{1, n^1}$. Then a BC for problem (1)-(2) is defined as an inequality of the form $\widehat{\alpha}^T x \le \widehat{\beta}_0, \widehat{\alpha}_j \in \{0,1\}$, $j = \overline{1, n^1}, \widehat{\beta}_0 = [\beta_0], \beta_0 = \widehat{\alpha}^T x^0$

The BC-generating inequality is $\varsigma^T x \leqslant \phi_0, \phi_0 = \varsigma^T x^0$, under the conditions $\varsigma_j = \sum\limits_{i \in I^B} \lambda_i a_{ij}$, $\lambda_i \geqslant 0$, where $a_{ij}, i \in I^B$ are the coefficients of the basis matrix and λ_i are the weights of the basis constraints. Each $\widehat{\alpha}^j$ is set in correspondence with the value $cs(\widehat{\alpha}^j) = \frac{\varsigma^T \widehat{\alpha}^j}{|\varsigma|_2 \left|\widehat{\alpha}^j\right|_2}, j = \overline{1, n}^1$. $cs(\widehat{\alpha}^j)$ defines how close each cut with the coefficients $\widehat{\alpha}^j$ is to the generating inequality (proximity measure).

Another important feature of BCs is their radicality measure r, which is defined as the number of unit hypercube vertices cut off by the BC, assuming that the cut is valid. For $\widehat{a}^T x \le b$, $b \in I_1^k, x \in I_{n^1}^2, \widehat{a} \in I_{n^1}^2$, $k = \sum\limits_{j=1}^{n^1} \widehat{a}_j, 1 \le k \le n^1$ and ${}^l_k = \frac{k!}{l!(k-l)!}$, we define: $r_k^b = 2^{n^1 - k} \sum\limits_{l=b+1}^{k} {}^l_k \rightarrow \max$.

Practical applications identified several classes of milp-reducible problems differing in the efficiency of the two measures in the BCBA. In problems allowing the construction of valid BCs, the use of the measure $cs(\widehat{\alpha}^j)$ allows the synthesis of valid cuts for on average 75percen of the algorithm steps, and the BCBA shows in experiments a speed that is statistically close to that of the polynomial algorithm [1]. However, there are classes of problems for which there are no valid BCs. Examples include makespan scheduling problems for parallel machines with delays in job entering and related problems [2]. In these cases, the radicality measure r proved to be much more efficient, leading, in some cases, to an increase in the speed of the BCBA computations by many orders of magnitude. The computational experiments also revealed the potential of using different BC construction strategies. The algorithm allows the use of a large menu of strategies with BCs that are best tailored to the structure of constraints (2).

References

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2. Avdeenko, T. V., Mesentsev, Y. A. : Efficient approaches to scheduling for unrelated parallel machines with release dates. IFAC Proceedings Volumes 49, iss. 12: 8 IFAC conference on manufacturing modelling, management and control MIM 2016, 1743–1748 (2016)