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OPTIMIZATION METHODS AND THEIR APPLICATIONS

30th of June – 6th of July, 2014
Olkhon Island, Baikal

Irkutsk
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Melentiev Energy Systems Institute SB RAS

Laboratory of Algorithms and Technologies for Network Analysis of National Research University «Higher School of Economics», Nizhny Novgorod

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The School-Seminar is dedicated to the 75th anniversary of Professor Oleg V. Vasiliev (1939-2002).

Oleg V. Vasiliev was Emeritus Professor of Irkutsk State University, he was mathematician in the field of optimal control and optimization well known and recognized. He was highly educated, experienced and qualified university lecturer. He was a well rounded bright and creative person, minded-open to new methods and tools in science, education and technology. He gained a number of honorary titles and degrees: Doctor of physical and mathematical sciences, Professor, Honored worker of Science and Technology of the Russian Federation, member of the American Mathematical Society , Honored Scientist of Buryat Republic; honored worker of Mongolian Public Education.

Administrative activities of Professor Vasiliev were marked by the following milestones: Head of the Department of Computational Mathematics of Irkutsk State University (ISU), the founder and head of the Optimization Methods Department of ISU, Dean of the Faculty of Mathematics of ISU, founder and director of the Institute of Mathematics and Economics of ISU, Chairman of PhD Examination Board of ISU. He received many awards and honors throughout his career including the Medal of the Order "For Service to the Motherland"of the 2nd class the Medal "For Valour"..

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PLENARY TALKS

TERMINAL CONTROL OF BOUNDARY MODELS[1]

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Consider the problem of extremal mapping: to find a fixed point $v^* \in W$, satisfying extreme inclusion

$$v^* \in \text{Argmin}\{\Phi(v^*, w) + \varphi(w) |\ w \in W \subset R^n\}. \tag{1}$$

Here $(v, w) \in W \times W \subset R^n \times R^n$, W is convex closed bounded set. The function $\Phi(v, w) + \varphi(w)$ is convex with respect to w for any $v \in W$. In regular case the extremal mapping (1) always has a fixed point. This statement includes many well-known finite-dimensional (static) problems, such as the convex or equilibrium programming problems, saddle-point problems, n-person games with the Nash equilibrium, multicriteria equilibrium problems, saddle-point games, economic equilibrium models, variational inequalities.

These problems are interpreted as mathematical models for objects from various areas of science and practice. It is assumed that the objects are immersed in the environment for which characteristics (properties) change over time. Changes to the environment is described by means of a controlled dynamical system. Objects and their mathematical models are also changing with the changing environment. In this situation, we consider the transfer an object from one state (initial) to another one (terminal) for a finite period of time.

$$v_0^* \in \text{Argmin}\{\Phi_0(v_0^*, v_0) + \varphi_0(v_0) \mid\ A_0 v_0 \le a_0\}, \tag{2}$$

$$v_1^* \in \text{Argmin}\{\Phi_1(v_1^*, v_1) + \varphi_1(v_1) \mid\ A_1 v_1 \le a_1, \tag{3}$$

$$\frac{d}{dt} v(t) = D(t)v(t) + B(t)u(t),\ t_0 \le t \le t_1, \tag{4}$$

$$v(t_0) = v_0^*,\ v(t_1) = v_1^*,\ u(t) \in U\}.$$

Here sets $W_i, i = 0, 1$ from (1) are given in the form of functional inequalities as constraints. Controls are taken from the ball of the Hilbert space $U = \{u(\cdot) \in L_2^r[t_0, t_1] \mid \| u(\cdot) \|_{L_2^r}^2 \le \mathrm{C}^2\}$, $D(t), B(t)$ -$n \times n, n \times r$ are continuous time-dependent matrix functions, A_0, A_1 are fixed matrices , a_0, a_1 are given vectors . If $\Phi_0(v_0^*, v_0) \equiv 0, \Phi_1(v_1^*, v_1) \equiv 0$, then (2),(3) is transformed into a convex programming problem . These models (2)-(4) allow us to describe transfer an object from one state to another one for a finite period of time.

Dynamical system (2)-(4) is considered in Hilbert space. It means that up to a set of measure zero, values of the function-control $u(\cdot)$ belong to the set $U \subseteq L_2^r[t_0, t_1]$. In the case where the control run through the entire set $u(\cdot) \in U$, differential system (4) generate a trajectories $v(t), t \in [t_0, t_1]$, which are absolutely continuous functions. Left $v(t_0) = v_0$ and right $v(t_1) = v_1$ ends trajectories describe the set of initial and terminal conditions . In the linear case , these sets are subspaces of R^n, which, in particular, may coincide with their spaces.

System (2)-(4) in the report, is treated as a convex-concave programming problem which is formulated on the direct product of the spaces $R^n \times L_2^n[t_0, t_1] \times L_2^r[t_0, t_1] \times R^n$. In this situation

[1]This work was supported by the RFBR (project no.12-01-00783) and by the Program "Leading Schools"(project NSh-4640.2014.1)

it is naturally to introduce two linearized Lagrangian (primal and dual), which have the form [1]

$$\mathcal{L}(v_0^*, v_1^*, p_0, p_1, \psi(\cdot); v_0, v_1, v(\cdot), u(\cdot)) = \langle \nabla_2 \Phi_0(v_0^*, v_0^*) + \nabla \varphi_0(v_0^*), v_0 \rangle$$
$$+ \langle p_0, A_0 v_0 - a_0 \rangle + \langle \nabla_2 \Phi_1(v_1^*, v_1^*) + \nabla \varphi_1(v_1^*), v_1 \rangle + \langle p_1, A_1 v_1 - a_1 \rangle$$
$$+ \int_{t_0}^{t_1} \langle \psi(t), D(t)v(t) + B(t)u(t) - \frac{d}{dt} v(t) \rangle dt, \tag{5}$$

and

$$\mathcal{L}^T(v_0^*, v_1^*, p_0, p_1, \psi(\cdot); v_0, v_1, v(\cdot), u(\cdot)) =$$
$$\langle \nabla_2 \Phi_0(v_0^*, v_0^*) + \nabla \varphi_0(v_0^*) + A^T p_0 + \psi_0, v_0 \rangle + \langle -a_0, p_0 \rangle$$
$$+ \langle \nabla_2 \Phi_1(v_1^*, v_1^*) + \nabla \varphi_1(v_1^*) + A^T p_1 - \psi_1, v_1 \rangle + \langle -a_1, p_1 \rangle$$
$$+ \int_{t_0}^{t_1} \langle D^T(t)\psi(t) + \frac{d}{dt}\psi(t), v(t) \rangle + \int_{t_0}^{t_1} \langle B^T(t)\psi(t), u(t) \rangle dt \tag{6}$$

for all $p_0, p_1 \in R_+^m$, $\psi(\cdot) \in \Psi_2^n[t_0, t_1]$, $(v_0, v_1) \in R^n \times R^n$, $(v(\cdot), u(\cdot)) \in AC^n[t_0, t_1] \times U$, where $AC^n[t_0, t_1]$ is the linear manifold of absolutely continuous functions from $L_2^n[t_0, t_1]$, $\Psi_2^n[t_0, t_1]$ is the class of absolutely continuous functions from $(L_2^n[t_0, t_1])^T$. The linear manifold $AC^n[t_0, t_1]$ is dense everywhere in $L_2^n[t_0, t_1]$, i.a. the closure of $AC^n[t_0, t_1]$ in norm $L_2^n[t_0, t_1]$ coincides with $L_2^n[t_0, t_1]$. Here v_0^*, v_1^* are parameters of the Lagrangians.

Both Lagrangians have the same saddle points $(p_0^*, p_1^*, \psi^*(\cdot); v_0^*, v_1^*, v^*(\cdot), u^*(\cdot))$. Components of them form primal $(v_0^*, v_1^*, v^*(\cdot), u^*(\cdot))$ and dual $(p_0^*, p_1^*, \psi^*(\cdot))$ solutions for problem (2)-(4).

The report presents the saddle iterative method for calculating the saddle point for both Lagrange functions. It is proved convergence of method to the saddle point for all components of the primal and dual solutions of initial problem (optimal) control [1],[2]. More precisely, the weak convergence is in respect controls, strong convergence is in the phase and dual trajectories, as well as for terminal saddle variables for boundary value problems corresponding to the ends of the time interval.

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SCATTERING AND INVERSE SCATTERING FOR A LEFT-DEFINITE STURM-LIOUVILLE PROBLEM

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This talk reports on a scattering and an inverse scattering theory for the Sturm–Liouville equation with a weight that changes sign, but with non negative potential. The crucial ingredient of the approach is a generalized transform built on the Jost solutions of the problem and hence termed the Jost transform and the associated Paley–Wiener theorem linking growth properties of transforms with support properties of functions. One motivation for this investigation comes from the Camassa–Holm equation for which the solution of the Cauchy problem can be achieved by the inverse scattering transform of a certain Sturm-Liouville problem.

ON EFFICIENTLY COMBINING LIMITED–MEMORY AND TRUST–REGION TECHNIQUES

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Limited memory quasi-Newton methods and trust-region methods represent two efficient approaches used for solving unconstrained optimization problems. A straightforward combination of them deteriorates the efficiency of the former approach, especially in the case of large-scale problems. For this reason, the limited memory methods are usually combined with a line search. We show how to efficiently combine limited memory and trust-region techniques. One of our approaches is based on the eigenvalue decomposition of the limited memory quasi-Newton approximation of the Hessian matrix. The decomposition allows for finding a nearly-exact solution to the trust-region subproblem defined by the Euclidean norm with an insignificant computational overhead compared with the cost of computing the quasi-Newton direction in line-search limited memory methods. The other approach is based on two new eigenvalue-based norms. The advantage of the new norms is that the trust-region subproblem is separable and each of the smaller subproblems is easy to solve. We show that our eigenvalue-based limited-memory trust-region methods are globally convergent. Moreover, we propose improved versions of the existing limited-memory trust-region algorithms. We presented results of numerical experiments demonstrating the efficiency of our approach which is competitive with line-search versions of the L-BFGS method.

DOMINATING SETS IN GRAPHS: OPTIMIZATION AND EXTENSIONS

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This talk will discuss several variations of dominating sets in graphs that are motivated by applications in wireless networks. Given a simple undirected graph, the minimum connected dominating set problem is to find a minimum cardinality subset of vertices D inducing a connected subgraph such that each vertex outside D has at least one neighbor in D. Approximations of minimum connected dominating sets are often used to represent a virtual routing backbone in wireless networks. We consider the following optimization problems dealing with dominating sets in wireless networks:

- The edge-weighted bottleneck connected dominating set problem, which seeks a minimum edge weight in the graph such that the corresponding bottleneck subgraph has a connected dominating set of size k [3, 5].
- The problem of finding a minimum dominating set of restricted diameter, which bounds the distance that information must travel through the network [1].
- A fault-tolerant version of the minimum dominating set problem that, in addition to restricting the diameter enforces a desired connectivity for the set [2].

In addition, we will show how the concept of domination can be used to develop efficient approximation algorithms and scale-reduction procedures for other important problems in wireless networks modeled as unit-disk graphs [4].

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UNIFIED MODELING AND THEORY FOR GLOBAL OPTIMIZATION

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Traditionally, global optimization problems in literature are usually formulated as

$$\min f(x) \ s.t. \ g(x) \leq 0,$$

where the «objective» function $f(x)$ and constraint $g(x)$ are assumed to be differentiable or simply Lipschitzian. It is well-known in nonlinear analysis [1] and mathematical physics [2] that a real-valued function is called objective only if it satisfies the frame-invariance principle [3]. However, in mathematical programming the objective function has been misused with other concepts such as cost, target, utility, and energy functions, etc [4]. Clearly, without detailed structural information on $f(x)$ and $g(x)$, it is difficult (or impossible) to have a general theory and effective methods for solving this artificially challenging problem. This could be one of main reasons why there has been no fundamental break-through in nonconvex programming over the past 50 years.

In this plenary lecture, the speaker will first present a unified mathematical model:

$$\min P(x) = W(Dx) + F(x) \ s.t. \ x \in X,$$

where D is a linear operator, $W(y)$ is an objective function, i.e. $W(y) = W(Qy)$, $\forall Q^T = Q^{-1}$, which depends only physical property of the system; while $F(x)$ is a «subjective» (or cost) function which depends each problem and must be linear. The feasible set X contains only linear constraints (boundary conditions). The speaker will show how the canonical duality-triality theory [2] is naturally developed, why this theory can be used not only for model complex systems within a unified framework, but also for solving a large class of nonconvex, nonsmooth, and discrete problems in both nonlinear analysis and global optimization. Some fundamental and conceptual mistakes in recent papers by C. Zalinescu and his co-workers will be revealed. Applications will be illustrated by a list of global optimal solutions to some well-known challenging problems in global optimization, nonlinear PDEs, and information technology [5,6], and in certain cases, the global minimal solution could be the worst decision.

This talk should bring some fundamentally new insights into complex systems theory, nonlinear optimization and computational science.

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OBTAINING EXAFLOPS PERFORMANCE FOR GLOBAL OPTIMIZATION PROBLEMS

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Great computational potential of modern supercomputer systems allow us to attempt solving many complex science and applied problems which was not able to analyze earlier. It is expected that supercomputer systems with the best performance will be substantially multiprocessors (up to billion computational cores), hybrid with various types and computational elements and multilevel structure for executing computations (distributed computational units - computational nodes with shared memory - multicore processor cores - computation accelerators). Effective utilization of huge computational potential of exaflops systems is a global "challenge"problem to all spectrum of computational sciences. Conditions for successful over-coming this problem are the following:
– significant time-consuming of solved problems (up to 10^{18} operations);
– high parallelism potential (scalability) of computation (up to use more that 10^9 processors);
– low intensity of information communications (locality) between parallel executed computations;
– computation stability to computational hardware failures which are unavoidable for so great number of computational elements.

It can be demonstrated that all given above conditions are fulfilled for global optimization problems.

In this paper the problems of multidimensional multiextremal optimization and parallel methods of their solution are considered. Only a general assumption is made regarding the optimizable function: the function is preset algorithmically (in the form of an algorithm of computation of values by input parameters) and satisfies the Lipschitz condition with an a priori unknown constant (problems of this kind are often found in applications). Within the limits of the considered approach solution of multidimensional problems is reduced to solution of one-dimensional problems equivalent to multidimensional problems. For problem dimension reduction a multi-level scheme is proposed, which combines the ideas of Peano-type space filling curves and nested optimization. The offered parallel algorithm uses a multi-level scheme of dimension reduction for effective parallelizing. Results of numerical experiments confirm convergence and speedup of the parallel algorithm in comparison with its sequential prototype.

BILEVEL OPTIMIZATION: MODEL, OPTIMALITY CONDITIONS AND FIRST SOLUTION APPROACHES [1]

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Considering the convex parametric optimization problem $\varphi(x) := \min_y\{f(x,y) : g(x,y) \leq 0\}$ with the solution set mapping Ψ, the bilevel optimization problem reads as

$$\text{"}\min_x\text{"}\{F(x,y) : G(x) \leq 0,\ y \in \Psi(x)\}. \tag{1}$$

Here $F, f, g_i, i = 1, \ldots, p$, are real functions mapping the $m \times n$-dimensional Euclidean spaces to the real line, and the functions $G_i, i = 1, \ldots, q$, are defined on the n-dimensional real space. First note, that problem (1) is not well posed in case that the set $\Psi(x)$ does not reduce to a singleton for all feasible x. In that case there are at least two possible ways out. Here we will use the optimistic approach $\min_{x,y}\{F(x,y) : G(x) \leq 0,\ y \in \Psi(x)\}$.

To solve this problem, it needs to be replaced with a one-level optimization problem. For that, at least two approaches are possible:

1. The use of the Karush-Kuhn-Tucker (KKT) conditions to replace the lower level problem. This leads to a mathematical program with complementarity constraints (MPCC) which, unfortunately, is not fully equivalent to (1), see Dempe and Dutta [1]. Necessary optimality conditions using this approach can be formulated for the bilevel optimization problem using ideas of nonsmooth analysis, see Zemkoho [2]. First algorithms for computing stationary solutions of problem (1) are presented. These algorithms have to respect that the Lagrange multipliers for the lower level problem need to be carefully selected such that a local optimum of the (MPCC) is really related to a local optimum of problem (1).

2. Replacing the lower level problem using the optimal value function $\varphi(x)$ leads to a nonsmooth (since the function $\varphi(\cdot)$ is not differentiable even under very restrictive assumptions) optimization problem

$$\min_{x,y}\{F(x,y) : G(x) \leq 0,\ g(x,y) \leq 0,\ f(x,y) \leq \varphi(x)\}.$$

   Partial calmness can be used to replace this problem using an exact penalty function approach. For the obtained Lipschitz optimization problem, necessary optimality conditions can be formulated using variational analysis again. Solution algorithms using this approach need to approximate the optimal value function, see e.g. Dempe and Franke [3] in the linear case.

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PROGRESS IN SOLVING THE COMPOUND MONOTONIC REGRESSON (CRM) PROBLEM

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The compound monotonic regression (CMR) problem arises in the context of testing psychological theories where the relationship between manifest variables (e.g., percent correct performance) and underlying latent variables (e.g., amount of learning) is assumed monotonic. If two manifest variables are products of the same latent variable, they will be comonotonic (i.e., monotonic functions of each other).

The CMR problem is to find the best-fitting values of a set of observations under comonotonicity (or isosign) constraints. That is, given vectors, $a, b, v, wn \in R^n$, and a set E of pairs (i, j) corresponding to edges of a directed acyclic graph, find x and y that minimize:

$$\sum_i v_i(x_i - a_i)^2 + \sum_i w_i(y_i - b_i)^2$$

Subject to:

$$\begin{aligned} x_i \leq x_j,\ \forall(i,j) \in E \\ y_i \leq y_j, \forall(i,j) \in E \end{aligned} \qquad (MR\ constrains)$$

And

$$\begin{aligned} x_i < x_j \Rightarrow y_i \leq y_j,\ \forall i,j,\ 1 \leq i,j \leq n \\ y_i < y_j \Rightarrow x_i \leq x_j,\ \forall i,j,\ 1 \leq i,j \leq n \end{aligned} \qquad (CMR\ constrains)$$

We present a branch-and-bound algorithm for solving this problem and examine its properties.

VARIATIONAL OPTIMALITY CONDITIONS WITH FEEDBACK DESCENT CONTROLS THAT STRENGTHEN THE MAXIMUM PRINCIPLE

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The talk is devoted to necessary optimality conditions for the optimal control problem (P):

$$\dot{x} = f(t, x, u), \quad x(t_0) = x_0, \quad u(t) \in U, \quad t \in T = [t_0, t_1], \tag{1}$$
$$J[x, u] = l(x(t_1)) \to \min .$$

Here, U is a given compact set. The function f is assumed to be continuous, locally Lipschitzian in x, and such that (s.t.) the set of all admissible trajectories of the control system is relatively compact in $C(T, R^n)$. The cost function l is locally Lipschitz continuous (for simplicity, below we assume that l belongs to the class $C^2(R^n)$).

Though problem (P) is stated within the conventional class $\mathcal{U} = L_\infty(T, U)$ of open-loop controls, the discussed optimality conditions are formulated in terms of feedback controls, which are arbitrary single-valued functions $v : T \times R^n \to U$. As concepts of a solution to (1) under a feedback control v we employ both Caratheodory solution concept, and Krasovskii-Subbotin constructive motions. Let $\mathcal{X}(v)$ denote the union of all solutions of these types. A control v is said to be a *descent control* (with respect to the functional J) at an admissible point $\bar{\sigma} = (\bar{x}, \bar{u})$ if there exists $x \in \mathcal{X}(v)$ s.t. $l(x(t_1)) < J[\bar{\sigma}]$. Clearly, optimality of $\bar{\sigma}$ in (P) implies the absence of descent controls at $\bar{\sigma}$.

In the considered optimality conditions, feedback controls, which are expected to be descent at a reference point $\bar{\sigma}$, are designed by means of an extremal aiming with an arbitrary support majorant of the functional J at $\bar{\sigma}$ (i.e., with a weakly decreasing solution $\varphi(t, x)$ of the respective Hamilton-Jacobi inequality with a certain boundary condition). In the simplest case, support majorants are defined by solutions of the adjoint system, while the respective optimality conditions provide a straightforward strengthening of the Maximum Principle. Below, we formulate such a strengthening for a non-smooth problem (P).

Let $\Psi(\bar{\sigma})$ denote the set of all solutions to the Clarke adjoint inclusion:

$$-\dot{\psi}(t) \in \partial_x[\psi(t) \cdot f(t, \bar{x}(t), \bar{u}(t))], \quad \psi(t_1) = l_x(\bar{x}(t_1)),$$
$$p(t, x) := \psi(t) + l_x(x) - l_x(\bar{x}(t)), \quad \psi \in \Psi(\bar{\sigma}),$$
$$U_\psi(t, x) := \operatorname*{Argmin}_{u \in U} p(t, x) \cdot f(t, x, u),$$

and let $\mathcal{V}_\psi$ denote the set of all selections of the multifunction $U_\psi(t, x)$.

Theorem (Feedback Minimum Principle). *Assume that a process $\bar{\sigma} = (\bar{x}, \bar{u})$ is optimal in problem (P). Then there exists $\psi \in \Psi(\bar{\sigma})$ s.t. the trajectory $\bar{x}$ is optimal in the following extremal problem*:

$$l(x(t_1)) \to \min; \quad x \in \bigcup_{v \in \mathcal{V}_\psi} \mathcal{X}(v).$$

As one can easily observe, this theorem encloses the non-smooth Maximum Principle by F. Clarke for the addressed optimal control problem. Note that, the proposed optimality condition is of a variational type, since it is formulated in terms of an auxiliary infinite-dimensional extremal problem. Moreover, further generalizations of these conditions with support majorants are of variation types as well. In the modern Hamilton-Jacobi Theory, these results are the first necessary conditions, which are comparable in efficiency with the Maximum Principle.

METHODS OF SOLVING THE STOCHASTIC PROGRAMMING PROBLEMS WITH QUANTILE CRITERION

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Many applied problems are described by notions of the stochastic programming (see [1]). The average value of a loss function is the traditional criterion for stochastic problems. However, this criterion does not correspond often to the real problem. For example, the similar situation arises when we should guarantee the high precision of the control with a given probability (see [2]). In this case, the quantile criterion (VaR-criterion) is more adequate (see [3]). But the quantile criterion is more complicated than the criterion in the form of the expectation.
The paper suggests two algorithms for solving a quantile optimization problem in the two-stage statement. A strategy is chosen at the first stage. After that some random event is realized, whose unfavourable influence is corrected by the second stage strategy. In this case, the quantile function is the optimality criterion. The special case is considered, in which the loss function is bilinear, i.e. the one is linear in the strategies and random vector, separately. The random vector has the normal distribution.
The first algorithm is based on discretization of Gaussian measure and on the reduction of the problem to a mixed integer linear programming problem. In order to decrease the computer time the confidence method (see [3]) and the notion of the probability measure kernel are used. The second algorithm is also based on the confidence method. But the problem is reduced to solving a sequence of convex programming subproblems which are parameterized by a scalar parameter. The Monte Carlo method and properties of Gaussian measure kernel are used to check the probabilistic constraints that arise in the process of solving the subproblems. Opportunities of both the algorithms are illustrated by an example.

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APPROXIMABILITY OF GEOMETRIC TRAVELING SALESMAN PROBLEM AND SOME ITS GENERALIZATIONS[1]

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We consider approximability issues for a series of combinatorial optimization problems, each of them generalizes the well-known traveliing salesman problem (TSP) and being a special subclass of vehicle routing problem simultaneously.

Basically, for a complete weighted graph, the optimal covering problem by k vertex-disjoint cycles (k-MHC) is studied. Optimization criterion is an additive one, it is required to find a cover having the minimum total weight of the belonging arcs. The problem has multiple applications, e.g. in cluster analysis and naturally generalizes traveling salesman problem (TSP) (because TSP is 1-MHC).

The general TSP problem is NP-hard and can not be approximated within 2^n (unless $P = NP$). Meanwhile, even metric TSP belongs to Apx. S.Arora has shown, that euclidean TSP (where vertices are points in some finite-dimensional euclidean space, and weights are euclidean distances among them) has polynomial-time approximation scheme (PTAS). That is, due to this shining result, the most popular subclass of TSP problem can be approximated efficiently in polynomial time within any given accuracy.

Our results are the following. We prove that k-MHC is strongly NP-hard for any fixed $k \geq 1$ and remains intractable in metric and euclidean cases. We propose 2-approximation polynomial-time algorithm for matric k-MHC and polynomial-time approximation scheme for euclidean k-MHC.

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SOLVING HUGE CONTINUOUS OPTIMIZATION PROBLEMS

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Many practical non-convex continuous optimization problems have a very large number of variables. Therefore, solution methods that are able to solve them exactly or approximately are very welcome. Recently we have suggested DE-VNS [1], a new heuristic that combined two well known meta-heuristic approaches: Differential Evolution (DE) and Variable Neighborhood Search (VNS). Both of them attracted a considerable attention by academics as well as by practitioners. In our hybrid heuristic, the idea of neighborhood change is used to estimate the crossover parameter of DE. Family of distributions are introduced, their parameters estimated during the search and then used to control the distances among solutions in the search space. It turned out that DE-VNS is more favorable than the recent DE approaches when tested on standard and large instances from the literature [2], such as Ackley and Rastrigin functions. In this talk I will show that DE-VNS is also able to solve exactly huge problems with up to 10,000 continuous variables. To the best of our knowledge, no attempt has been made so far to solve problems with such a huge number of continuous variables. In order to measure both the method quality and the instance hardness, we introduce terms of dimensional convergence and the dimensional stability. To the best of our knowledge no attempt has been made to evaluate the behavior of a method in as many as 100,000 dimensions, thus rendering our work as an initial one in the area of extremely large scale continuous optimization.

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ON GRAPH-THEORETICAL APPROACH TO SOLUTION OF LARGE-SCALE OPTIMIZATION PROBLEMS [1]

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In this report the problem of solving a large-scale standard linear programming problem $\min cx : Ax = b,\ x \geq 0$ with sparse but unstructured constrain matrix A is considered. This problem is reformulated as the orthogonal projection problem on the feasible polyhedron and further on reduced to the the projection of a certain fixed point onto the convex cone generated by rows of matrix A.[1].

The projection problem becomes much easier if the constrain matrix is decomposed with the help of coloring problem for the graph of constraint connectivity. The vertexes of the graph of the same color will correspond to the set of structurally-independent constraints, and projection onto such set has linear complexity with respect to the cardinality of this set. At the same time the number of such sets is not too large due to the sparsity of the original problem. Despite the fact that the coloring problem is NP-hard [2], the numerous heuristics for finding for instance maximal independent sets can be used for its satisfactory solution [3].

The talk is devoted to the current state of the implementation of this approach with especial attention given to the coordination problem to link the subset projections and the applications of this approach in transportation modeling and logistics.

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OPTIMIZATION AND ECONOMIC MODELING OF ENERGY SYSTEM CENTERING CO2 ISSUES

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Global warming has become a critical issue to our planet, Earth. The continuing rise in the average temperature of the atmosphere and ocean causes serious problems such as: glacier shrinkage, sea level elevation, species extinction, agricultural output decrease, and extreme weather events. Many countries, territories, and international organizations have joined together to curb the global warming trend. The United Nations Framework Convention on Climate Change (UNFCCC) began in 1992 and Kyoto protocol, initiated in 1997. Among the many efforts, curbing Carbon Dioxide (CO2) emission has been an important one.

Along this line, research efforts to incorporate CO2 in energy systems operations have surged in recent years. This talk will review the mathematical modeling, optimization techniques, and economic analysis of CO2 in energy systems. This will include CO2 policy and market modeling, integration of CO2 mitigation into power systems' operations and strategic planning. On the other side, as the silver bullet for solving CO2 emission problems, carbon capture and storage will be discussed by showing the techno-economic analysis and related optimization models.

RECENT DEVELOPMENTS IN LIPSCHITZ GLOBAL OPTIMIZATION[1]

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In this lecture, the global optimization problem of a multidimensional function satisfying the Lipschitz condition over a hyperinterval with an unknown Lipschitz constant is considered. It is supposed that the objective function can be "black box", multiextremal, and non-differentiable. It is also assumed that evaluation of the objective function at a point is a time-consuming operation. Many algorithms for solving this problem have been discussed in literature (see [1–8] and references given therein). They can be distinguished, for example, by the way of obtaining information about the Lipschitz constant and by the strategy of exploration of the search domain. Different exploration techniques based on various adaptive partition strategies are analyzed.

The main attention is dedicated to two types of algorithms: (i) methods using space-filling curves to reduce the dimensionality of the global optimization problem; (ii) diagonal global optimization algorithms. Both classes of methods have a number of attractive theoretical properties and have proved to be efficient in solving applied problems. Several families of deterministic derivative-free numerical algorithms are discussed. A number of unconventional ideas, such as adaptive strategies for estimating Lipschitz constant, balancing global and local information to accelerate the search, etc. are presented. Results of extensive numerical experiments performed on the GKLS-generator (see [1]) demonstrate advantages of the proposed methods with respect to a number of traditional algorithms.

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USE OF INTEGRAL EQUATION APPARATUS IN INFRARED TOMOGRAPHY[1]

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In the report, the known technique of IR tomography of a hot gas [1] on the example of the flame of the widely used laboratory burner is newly stated. The variant is described for using two regimes of flame diagnosis: the active one (ON) – with included translucence source and passive one (OFF) – without such source. The active-passive diagnosis makes possible to obtain two experimental functions and two new two-dimensional integral equations concerning the absorption coefficient k and the Planck function B (it describes the emission and gives the possibility to calculate the temperature profile T of a medium).

In the case of axial symmetry and parallel scanning the flame, the two-dimensional equations are transformed into one-dimensional singular integral equations (SIE) of Abel's type concerning k and B. For its numerical solving, one uses the generalized quadrature method [2] (in a new version) and the Tikhonov regularization method with experimental data smoothing by splines and without smoothing.

The software package for MatLab 7 is developed. With its help, results of experimental diagnosis of the burner flame for some wave numbers ν and in some flame cross-sections were processed. This processing confirms that a SIE has the self-regularization and its solution problem is moderately ill-posed. Use of the generalized quadrature method with preliminary spline-smoothing the experimental data, but without the Tikhonov's regularization gives the possibility to obtain practically the same results as with use of the Tikhonov regularization method.

Specific feature of the proposed technique is that it does not require a special determination of the absorption coefficient k by a direct measuring or through a database, for example, the HITRAN/HITEMP [3].

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CONSTRAINED GLOBAL OPTIMIZATION: MULTIPROCESSOR COMPUTATIONS

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Problem of multidimensional global optimization of a Lipschitzian function defined on the N-dimensional hypercube can be reduced to solving a one-dimensional problem over the interval [0,1] by applying a Peano space-filling curve mapping the hypercube onto the interval [0,1] of real axis (see [1,4]). For solving the one-dimensional problem there is a set of information-statistical algorithms for global search oriented at multiprocessor computations and implementing efficient parallelization [1-3].

A new approach having the potential of mass parallelism is to use for reducing multidimensional problem a set of space-filling curves (multiple mappings) [3,4]. It allows to use for solving the global optimization problem many thousands of processors and to speed up computations significantly.

This approach can be spread to global search problems with complicated Lipschitzian constraints on the base of applying the index method of constraints reduction [1]. The index method requires (as opposed to the classic penalty function method) no choice of parameters (like penalty constant), enables to solve optimization problems with multiextremal partially defined constraints and provide the possibility of significant parallelization.

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VARIATIONAL MAXIMUM PRINCIPLE IN CONTROLLED SYSTEMS OF MULTI-DIMENSIONAL HYPERBOLIC EQUATIONS [1]

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A problem of optimal control over hyperbolic systems of first-order multi-dimensional differential equations with distributed control is considered. Basing on multi-dimensional analogues of the Riemann invariants first introduced in papers [1,2] a necessary optimality condition in form of variational maximum principle is obtained. One-dimensional version of this optimality condition for semi-linear hyperbolic systems was published in paper [3].

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ON EXISTENCE OF CORE IMPUTATIONS IN SOME CLASSES OF FUZZY TU-COOPERATIVE GAMES[1].

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In the paper, we analyze so-called $\mathcal{S}$-representation of fuzzy TU-cooperative game. By applying this representation we facilitate essentially investigation of the core imputation existence problem. Special attention is paid to the games with rather simple structure of their $\mathcal{S}$-representation. In particular, some new analogs of superlinearity are obtained that provide nonemptyness of the core for the continuous homogeneous fuzzy games.

Introduced by the author, $\mathcal{S}$-representation v^* of a fuzzy n-person TU-cooperative game $v:\ [0,1]^n \to \mathbf{R}$ is defined by the formula

$$v^*(\tau^*) := \sup\{v(t\tau^*)/t \mid t \in (0, 1/\parallel \tau^* \parallel_\infty]\}, \quad \tau^* \in \mathcal{S},$$

with $\mathcal{S}$ to be a standard simplex $\mathcal{S} := \{\tau \in [0,1]^n \mid \sum_{i=1}^n \tau_i = 1\}$, and

$$\parallel \tau \parallel_\infty := \max\ \{|\tau_i| \mid i = 1, \ldots, n\} \quad \tau = (\tau_1, \ldots, \tau_n) \in \mathbf{R}^n$$

(for more details, see [1,2]). One of the main results of the paper is the following criterium of nonemptyness of the core.

Theorem. *Let v be a continuous and homogeneous fuzzy n-person TU-cooperative game. Then there exists a core imputation of this game if and only if for each $g \in \mathbf{R}^n$ such that $\sum_{i=1}^n g_i = 0$ and $\parallel g \parallel_\infty \leq 1/n$ it holds*

$$v(c^*) \geq \frac{1}{2}[v(c^* + g) + v(c^* - g)],$$

with $c^* = (1/n, \ldots, 1/n)$ to be a barycenter of $\mathcal{S}$.

In conclusion, we apply the results obtained to the core imputation existence problem for several types of fuzzy cooperative games arising from concrete game-theoretic settings (airport games, linear production games, etc.)

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OPTIMAL CONTROL METHODS FOR CONSERVATION BIOLOGY: A CASE OF NON-HARVESTING UTILITY[1]

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The main purpose of this paper is to retrace the evolution of mathematical models focused on relation and interaction between economic growth, sustainable development and natural environment conservation. The starting point is a simple model of common-property harvesting, where renewable resource grows according to the course of nature. Further, this model is amended with defensive expenditures that favor the species growth. Apart from solely harvesting models, a transition model comprising both harvesting and non-harvesting values of wild biological species is presented. Preponderantly, all these models are designed to seek for long-term optimal and/or sustainable strategies for harvesting, where species preservation guarantees the profit stability for future generations and thus contributes to the economic development.

On the other hand, there is a group of purely non-harvesting models where anthropic activities and economic growth may have positive or negative impact on the natural evolution of wild species. Several scholars have proved that optimal strategies that are relatively good for harvesting purposes are not merely transferrable to the context of conservation of wildlife biological species with no harvesting value. However, existence of long-term conservation policies for all biological species (with or without harvesting value) cannot be guaranteed without having relatively large species populations at initial time. Therefore, all such strategies are incapable to enhance scarce population of endangered species and save them from eventual (local) extinction.

As an alternative, policy makers are compelled to design and implement short-term defensive actions aimed at enhancement of wildlife species populations. The latter is referred to as an emergent area of research in conservation biology.

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SECTION TALKS

DISCRETE OPTIMIZATION

MODEL SHEET RECTANGULAR PACKING PROBLEM

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The paper considers the model of the sheet rectangular packing problem of the set of rectangles with a given sizes $a_i \times b_i$, $i = 1 \dots n$ $(a_i \geq b_i)$ on the sheet with given size $A \times B$. As the origin selects the lower-left corner of the sheet. Variables of the model are the coordinates placement rectangles on the sheet, x_i, y_i $i = 1 \dots n$, and three sets boolean variables: z_i $i = 1 \dots n$ (the orientation of i-th rectangle along or across a sheet), s_{ij}, t_{ij} $i = 1, \dots, n-1$ $j = i+1, \dots, n$ (used for the formulation of conditions of no intersections of ith and j-th rectangles on horizontal and vertical). The model is represented as the following system of linear inequalities with partial boolean variables:

$$x_i \geq 0, x_i + (b_i - a_i)z_i \leq A - a_i, \quad i = 1, \dots, n, \tag{1}$$

$$y_i \geq 0, y_i + (a_i - b_i)z_i \leq B - b_i, \quad i = 1, \dots, n, \tag{2}$$

$$-x_i + x_j - (b_i - a_i)z_i + At_{ij} + As_{ij} \geq a_i, \quad i = 1, \dots, n-1, \quad j = i+1, \dots, n, \tag{3}$$

$$x_i - x_j - (b_j - a_j)z_j + At_{ij} - As_{ij} \geq a_j - A, \quad i = 1, \dots, n-1, \quad j = i+1, \dots, n, \tag{4}$$

$$-y_i + y_j - (a_i - b_i)z_i - Bt_{ij} + Bs_{ij} \geq b_i - B, \quad i = 1, \dots, n-1, \quad j = i+1, \dots, n, \tag{5}$$

$$y_i - y_j - (a_j - b_j)z_j - Bt_{ij} - Bs_{ij} \geq b_j - 2B. \quad i = 1, \dots, n-1, \quad j = i+1, \dots, n, \tag{6}$$

$$z_i \in \{0, 1\}, \quad i = 1, \dots, n, \tag{7}$$

$$s_{ij} \in \{0, 1\}, \quad i = 1, \dots, n-1, \quad j = i+1, \dots, n. \tag{8}$$

$$t_{ij} \in \{0, 1\}, \quad i = 1, \dots, n-1, \quad j = i+1, \dots, n. \tag{9}$$

Restrictions (1), (2) define the conditions of accommodation rectangles on the sheet and restrictions (3)–(6) define the conditions pairwise no intersections rectangles. Detail conclusion of the model (1)–(9) is given in [1].

If to add to the model (1)–(9) objective zero function to find its solution, we can use the Land and Doig method, which is the branches and borders method and is intended for solving the partially integer linear programming problem. Stop calculations made after receiving the first valid solution.

Numerical experiments showed practical applicability of the model for solving problems with count of rectangles less 15.

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HEURISTIC FOR THE TRUCK AND TRAILER ROUTING PROBLEM [1]

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In this talk we present a multi-start greedy heuristic for the Heterogeneous Fleet Truck and Trailer Routing Problem (HFTTRP) arising in a real-life practice of delivering goods from a warehouse to stores. One of the first papers devoted to the HFTTRP problem with Time Windows (HFTTRPTW) is the paper of Semet & Taillard (1993). Later Hoff & Lokketangen (2007) developed a tabu search algorithm for the HFTTRP problem with multiple depo arising in milk collection. Caramia & Guerriero (2010) suggested a multi-start 3-stage heuristic for a similar problem with one depot. Drexl (2011) developed an integer programming model and a branch-and-price algorithm for the HFTTRPTW problem.

In our talk we consider the HFTTRPTW problem with restrictions of stores on the vehicles which can perform delivery, with soft and hard time windows, and with splitting a delivery of one store to two or more vehicles. To our knowledge the problem has never been considered in the literature in such a general formulation.

Our heuristic is based on a greedy insertion of stores to a route similar to the greedy algorithms suggested by Solomon (1987). Additionally we apply an improvement procedure which moves all the deliveries in the current route to earlier time in order to avoid delays. The algorithm of building a complete solution is iteratively repeated many times, and every new route is started from one of the farthest stores chosen randomly. The computational experiments on real-life data including up to 400 orders and 100 vehicles demonstrate the efficiency of the suggested heuristic.

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CONSTRUCTION OF CYCLIC SCHEDULING WITH DUPLICATED MACHINES[1]

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In this paper we consider the situation where N identical jobs have to be processed on assembly line with m types of machines. There is technological route for job with n successively processing operations $O_1, O_2, \ldots, O_n$. Each operation O_j is described by a processing time p_j and machine m_j, $j = 1, 2, \ldots, n$. Machines in technological rout can repeat. Interruption of operations are forbidden. Simultaneously processing of two or more operations on one machine is impossible. Schedule in which the same tasks for different jobs begin processed through constant time period, named *cyclic*. Need to build permissible schedule with minimal cycle length. This is polinomial problem with cycle length equal to sum of tasks time for the most loaded machine.

Sometimes in practice one of tasks O_l takes much more time than other ones. In this case it is possible to make cycle length shorter if we begin to use more machines of m_l type. So we want to minimize cycle length C for schedule of processing N identical jobs and ability to use simultaneously several parallel machines for some types of machines. This problem is also polinomial.

The other important property of cyclic schedule is H – count of simultaneously processed jobs. If the number of simultaneously process jobs limited by value H, than the problem is NP-hard [1], the pseudopolinomial algorithm is known for fixed H [2].

In this paper we represent an algorithm for build cyclic schedule of processing N identical jobs with parallel machines and limited number of simultaneously processed jobs with minimal cycle length. The algorithm is based on dynamical programming and it laboriousness is $O((2P)^{2H-1}H^H e^H)$, where $P = \sum\limits_{j=1}^{n} p_j$.

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PROCUREMENT OPTIMIZATION WITH POSSIBILITY OF ALTERNATIVE USE OF CAPITAL [1]

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Company buys goods on the stock exchange and sells them. Some values and functions are defined: λ is the intensity of the goods sale, $\alpha + \beta v$ are the costs for the purchase and delivery of goods in the amount of $v > 0$, c is the selling price of one good, c_{xp} are specific storage costs. Periodicity T and purchase size $v = T\lambda$ are to be found to maximize the profit. Problem without storage costs was considered in [1].

We use discounting technology to compare costs in different time. Let r_0 be the rate of alternative risk-free capital allocation. Intensity of the incomming cashflow $c\lambda$ with discounting is $\frac{c\lambda}{(1+r_0)^t}$. Income discounted to the initial time for period T is

$$\int_0^T \frac{c\lambda}{(1+r_0)^t} dt = \frac{c\lambda}{ln(1+r_0)} \cdot \left(1 - \frac{1}{(1+r_0)^T}\right).$$

Storage costs in the delivery moment

$$\int_0^T \frac{(T\lambda - t\lambda)c_{xp}}{(1+r_0)^t} dt = \frac{c_{xp}\lambda}{ln^2(1+r_0)} \cdot \left(T \cdot \ln(1+r_0) + \frac{1}{(1+r_0)^T} - 1\right).$$

The profit to be maximized is

$$\frac{1}{T}\left(\frac{c\lambda}{ln(1+r_0)} \cdot \left(1 - \frac{1}{(1+r_0)^T}\right) - \alpha - \beta v - \frac{c_{xp}\lambda}{ln^2(1+r_0)} \cdot \left(T \cdot \ln(1+r_0) + \frac{1}{(1+r_0)^T} - 1\right)\right).$$

This function is convex and has a unique maximum. Model can be generalized to the case of irregular deterministic demand, but there is no periodicity. It is needed to optimize profits on the fixed interval with the boundary constraints of supply of the product. Discrete model is built, dynamic programming algorithm is proposed. Approaches to profit maximization in multiproduct systems are described.

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THE OPTIMAL SEGMENTATION OF GRAPH

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We investigate the following problem which is associated with segmentation graph.

SEGMENTATION GRAPH PROBLEM (or briefly SGP)

Instance: A connected graph $G = (V, E)$, $|V| \geq 2$, $|E| \geq 1$; a non-negative integer $K \leq |V|$; a positive integer $L \leq |E|$.

Question: Is there a set $B \subseteq V$ of cardinality K, which segmentation graph $G = (V, E)$ on the set B generates a set of segments $\mathfrak{R} = \{G_1, G_2, \ldots, G_p\}$ and $w(G) = \max\{|E_i| : 1 \leq i \leq p\}$ is the maximum (by number of edges) size of segment $G_i = (V_i, E_i) \in \mathfrak{R}$?

Under segmentation graph $G = (V, E)$ on the set $B \subseteq V$ we understand a partition of the set of edges of G, that two edges belong to the same segment of G_i, if and only if in the graph G_i exists (a, b)-path, that includes both of these edges and no contains vertices belonging to the B, except possibly vertices a and b. The set of vertices V_i of segment $G_i = (V_i, E_i)$ comprises end vertices of edges belonging to E_i.

Similar formulations of the SGP studied previously in [1, 2] for the design of trunk pipeline networks. It is known that such problems are NP-hard.

In this paper, we continued to study the SGP. We offer operation segmentation graph $G = (V, E)$ on the set $B \subseteq V$, which indicates constructively as produce different segmentation. We showed that a set of segments $\mathfrak{R} = \{G_1, G_2, \ldots, G_p\}$ connected graph $G = (V, E)$, where $|V| \geq 2$, $|E| \geq 1$, uniquely determined by $B \subseteq V$. For a fixed B the set $\mathfrak{R}$ can be constructed in time $O(|V| + |E|)$. We have proved the properties of segments that show design features of admissible and optimal solution. Presented results can be used to develop algorithms for solving SGP.

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EXPERIMENTAL STUDY OF AN EXPONENTIAL NEIGHBORHOOD FOR THE SERVERS LOAD BALANCING PROBLEM

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We consider the servers load balancing problem which is originated from optimal web hosting in cloud computing. We are given a set of servers. Each server contains the set of disks (images of disks). Internet sites with heterogeneous information are distributed among the disks. A lot of users visit these sites and generate the load for the servers. The load is changed during time interval and can be characterized by some parameters: CPU, RAM and others. We assume that the activities of users are known for each site and as result for each disk and each server. If the load of the server does not exceed a given threshold, we say that the server is working in regular mode. Otherwise, it works in heavy mode. To avoid the heavy mode we can move the disks from one server to another. This moving requires some computational efforts. We will call them as overhead expenses. We assume that the expenses are known for each parameter if a disk is extracted from one server and moving to another one. Initial allocation of disks among the servers is given. The problem is to find a new allocation of the disks to minimize the total exceeding over the thresholds during the time interval subject to overhead expenses for each server.

We develop a new exponential neighborhood in order to tackle this problem. Elements of this neighborhood are optimal solutions of the assignment problem, which is constructed as follows: we eject one disc from each server and redistribute these discs optimally. We study different disk ejection strategies and exploit the behavior of the local search algorithms based on the randomized version of this neighborhood. Computational results for 20 servers and 200 disks are discussed. Comparison with results from [1] is presented.

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METAHEURISTICS FOR THE SERVERS LOAD BALANCING PROBLEM[1]

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We consider the following problem arising in the optimization of cloud hosting. Heterogeneous Internet sites collected in structures called discs. The discs are distributed between the available set of servers. User inquiries to the site create a certain load on the server that stores the disc with this site. The load is characterized with a number of parameters and changes over time. Time is assumed to be discrete. The value of total load produced by all sites of each disc is known at each moment of time and for each parameter.

If the server's load exceeds known threshold value for some parameter, the server fails to handle requests in time. To avoid the overload discs can be moved from one server to another. Such movement requires some computational costs, which we call overheads. Overheads for ejection of the disc from the server and insertion of the disc to the server are known. The problem is to change known initial distribution of the discs so as to minimize total overload throughout the planning period with respect to overhead constraints.

To our knowledge, the first time the problem was considered in [1], where it is shown that the problem is NP–hard. To solve it the heuristic algorithm based on the linear relaxation is proposed in [1] as well. In this work we propose and compare three approaches based on ideas of stochastic local search, genetic algorithm, and variable neighborhood search. Computational experiments with randomly generated instances show, that proposed methods are significantly outperform proposed in [1] one in terms of accuracy and speed.

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APPROACHES TO SOLUTION UNIVERSITY TIMETABLING

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We consider two problems of university timetabling. The first problem is timetabling in non-Russian universities where individual study programs of students are taken into account(Course Timetabling). The second problem is timetabling in Russian universities where students are considered in groups. It is an expanded combination of problems School Timetabling when availability of the rooms isn't considered, all groups are engaged in own audiences and don't change them and Course Timetabling when availability of rooms is considered and in a room can be different groups. The university timetabling is NP-Complete [1].

We consider 4 approaches to solve the problems:

- harmony search [1];
- non-linear great deluge [2];
- honey-bee mating optimization [3];
- branch-and-cut [4].

Result of comparison are presented with a conclusion of their applicability in university timetabling software.

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POST-OPTIMAL ANALYSIS OF A VECTOR BOOLEAN PORTFOLIO OPTIMISATION PROBLEM WITH EXTREME OPTIMISM CRITERIA[1]

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We consider s-criterial discrete variant of Markowitz's investment problem [1] with extreme optimism criteria:

$$\max_{x\in X}\ \max_{i\in N_m}\sum_{j\in N_n} e_{ijk}x_j, \quad k\in N_s=\{1,2,\dots,s\}, \tag{1}$$

where e_{ijk} be an expected assessment of efficiency (yield) of measure $k\in N_s$ of investment project j in the situation when the market is in state i; $X\subset \mathbf{E}^n=\{0,1\}^n$ be a set of all admissible investment portfolios $x=(x_1,x_2,\dots,x_n)$, where $x_j=1$ if the project $j\in N_n$ is implemented, and $x_j=0$ otherwise. Pareto set of the investment problem (1) is denoted by $P^s(E)$, where $E=[e_{ijk}]\in \mathbf{R}^{m\times n\times s}$. We define arbitrary Hölder norm l_p, $1\le p\le\infty$, in spaces $\mathbf{R}^n$ and $\mathbf{R}^s$, and the Chebyshev norm l_∞ in the space $\mathbf{R}^m$, i. e. by the norm of a matrix $E\in\mathbf{R}^{m\times n\times s}$ is meant $\|E\|_{p\infty p}=\|(\|E_1\|_{p\infty},\|E_2\|_{p\infty},\dots,\|E_s\|_{p\infty})\|_p$, where $\|E_k\|_{p\infty}=\|(\|e_{1k}\|_p,\|e_{2k}\|_p,\dots,\|e_{mk}\|_p)\|_\infty$, $k\in N_s$. Here $E_k\in\mathbf{R}^{m\times n}$ is k-th cut of matrix E, $e_{ik}=(e_{i1k},e_{i2k},\dots,e_{ink})$ is i-th row of that cut. As usual [2], the stability radius $\rho(m,n,s,p)$ of the problem (1) is defined as $\sup\Xi(p)$ if $\Xi(p)\ne\emptyset$. Otherwise we consider $\rho(m,n,s,p)=0$. Here $\Xi(p)=\{\varepsilon>0:\forall E'\in\Omega(p)\quad (P^s(E+E')\subseteq P^s(E))\}$, $\Omega(p)=\{E'\in\mathbf{R}^{m\times n\times s}:\|E'\|_{p\infty p}<\varepsilon\}$.

Theorem. *For $P^s(E)\ne X$, any $m,n,s\in\mathbf{N}$ and $p\in[1,\infty]$ for the stability radius the following bounds are true*

$$\varphi\le\rho(m,n,s,p)\le(ns)^{1/p}\psi,$$

where

$$\varphi=\min_{x\notin P^s(E)}\ \max_{x'\in X(x,E)}\ \min_{k\in N_s}\frac{f_k(x')-f_k(x)}{\|x'\|_q+\|x\|_q},\quad \psi=\min_{x\notin P^s(E)}\ \max_{x'\in X(x,E)}\ \min_{k\in N_s}\frac{f_k(x')-f_k(x)}{\|x'-x\|_1},$$

$$X(x,E)=\{x'\in P^s(E):f(x')\ge f(x)\quad\&\quad f(x')\ne f(x)\},\quad f(x)=(f_1(x),\dots,f_s(x));$$

$$f_k(x)=\max_{i\in N_m}\sum_{j\in N_n}e_{ijk}x_j,\ k\in N_s;\quad 1/p+1/q=1.$$

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ON PROJECT SCHEDULING PROBLEM WITH CHANGING CONSUMPTION AND SUPPLY RATES OF RENEWABLE RESOURCES[1]

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We consider NP-hard project scheduling problem under resource constraints. Let the project consists of a set of interrelated tasks $I = \{1, \ldots, m\}$. The interrelation between tasks is defined by the sequence relation of a kind $i \to j$, where task j can not start before completion of task i. Preemptions of tasks are not allowed. There are n kinds of renewable resources that may be used to perform a task. Each task $i \in I$ is characterized by its execution time $p_i \in \mathbf{Z}^+$ and rates of resource consumptions, that may change in execution time of the task and are defined in the following way. The execution time of task $i \in I$ is divided into a_i^q periods of integer duration for each $q = 1, \ldots, n$. The consumption rate of resource of kind q by the task is constant in each period. At different time of planning horizon the resource availability of each kind may be different. There are $b_{\max}^q$ periods of integer duration, in each of that the availability of resource of kind q is constant, $q = 1, \ldots, n$.

It is required to find a schedule, which satisfies the partial order of processing of tasks and restrictions on resource consumptions, minimizing the total completion time $C_{\max}$.

The problem with single renewable resource has been considered in [1, 2]. Integer linear programming models have been formulated, dynamic programming algorithms have been developed and some polynomially and pseudo-polynomially solvable cases have also been identified.

In this work we generalize dynamic programming algorithms [1, 2] on the case, when there are several kinds of resources. The basic idea of proposed algorithms consists in enumeration of every possible state of processing of tasks, and for each state the minimal time moment, when it is reached, is calculated. The problem is proved to be NP-hard, but pseudo-polynomially solvable if the width of the partial order is bounded by a constant. New polynomially solvable cases of the problem are found.

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MINIMIZING THE NUMBER OF EQUAL SECTOR IN REGULAR COVERING OF THE PLANE[1]

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In regular coverages plane domain is divided into equal regular polygons (tiles), and all the tiles are covered equally with various figures [1-3]. In this paper, as the tiles used an equilateral triangle, and as the figures — identical sectors.

The problem of constructing a regular cover of the plane with the same sectors in which the number of sectors per unit of area coverage is minimal is considered. This problem is close to the problem of constructing the least dense cover (coverage density is the ratio of areas of figures to the area of covered domain) [4-6] but does not coincide with it completely.

Studied various models for optimal coverage of an equilateral triangle with one sector, which depend on the angle of the sector. The results obtained allow to find the optimal number of sectors for the various angular sectors which cover one tile in the case when each sector involved in covering only one tile, and the nodes of sectors covering one tile are at a single point.

Similar results are obtained for coverage using regular tiles in the form of square and a regular hexagon. A comparative analysis of coverage models with different tiles was performed depending on the sector angle. However, recent results are not included in this report.

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ALGORITHMS WITH PERFORMANCE GUARANTEES FOR SOME HARD PROBLEMS ON GRAPHS [1]

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The report gives an overview of some results obtained for the past five years by the author and his colleagues at the Institute of Mathematics SB RAS. It's about developing efficient algorithms with performance guarantees for solving such hard problems of discrete optimization and operations research as the routing problem, the multi-index planar assignment problem, clustering problem, facility location problem on graphs and networks, etc. Often discrete optimization problem on a graph associated with finding a subgraph of extreme total weight. Some of these problems are solvable in polynomial time, for example, the problem of finding the minimum-weight spanning tree problem. Examples of intractable problems of this kind are the traveling salesman problem and searching clique of a given size. Recently time attention has been paid to the study of problems in which you want to find in the complete weighted graph several edge-disjoint objects (such as a Hamiltonian cycle, substitution, cliques, spanning trees). Some of these extended problems remain effectively solvable (eg, the problem of finding multiple edge-disjoint spanning trees of minimum total weight). However, most of problems are NP-hard, and because to address them is urgent to develop effitient (polynomial) algorithms with the performance guarantees. Some examples of such algorithms are given in [1-5].

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EFFICIENT ALGORITHMS WITH PERFORMANCE GUARANTEES FOR SOME PROBLEMS OF FINDING SEVERAL CLIQUES IN A WEIGHTED COMPLETE UNDIRECTED GRAPH[1]

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Consider the following m-**Weighted Clique Problem (m-WCP).**

Input: a weighted complete undirected graph $G = (V; E; a; c)$, where $a\colon V \to \mathbb{R}$, $c\colon E \to \mathbb{R}$, and positive integers $L_1, \ldots, L_m$ such that $\sum_{i=1}^{m} L_i \leqslant n$.

Output: a family of disjoint cliques in the graph G of orders $L_1, \ldots, L_m$ with the smallest total weight of vertices and edges contained in these cliques.

We show that the problem is NP-hard in a strong sense and inapproximable in the general case and in some of its important special cases. We propose an approximation algorithm with time complexity $O(n^{m+2} \log n)$ for the case when the weights of vertices of the graph G are nonnegative and the weights of edges either satisfy the triangle inequality or are squared pairwise distances between some points of the Euclidean space.

The idea of approximation for m-WCP is based on the use of the exact solution of the special (auxiliary) problem of finding m disjoint stars of corresponding orders. It is shown that the exact solution of this problem can be find in time $O(n^{m+2} \log n)$ using the transportation algorithm. This algorithm is polynomial if m is fixed. The proposed approximation algorithm for the problem m-WSP has the same time complexity. It is proved that for the Metric m-WCP we have the approximation guarantee

$$2\Big(1 - \frac{\sum_{i=1}^{m} S(B_i^*)}{\sum_{i=1}^{m} L_i S(B_i^*)}\Big),$$

where $S(B_i^*)$ is the total weight of the vertices and the edges in the i-th star of the auxiliary problem, $i = 1, \ldots, m$. For the Quadratic Euclidean m-WCP the approximaton ratio 2 is proved.

The present paper generalizes and deepens the results obtained in [1]. We hope that the proposed approach can be extended for other actual subclasses of m-WCP.

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ON 2-CAPACITATED PERIPATETIC SALESMAN PROBLEM[1]

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The undirected m-Capacitated Peripatetic Salesman Problem (m-CPSP) [3] is a problem of finding m Hamiltonian cycles of least or greatest total cost in a given graph $G = (V, E)$, where each edge $e \in E$ has a cost c_e and an capacity C_e, such that no edge e is used more than C_e times. Edge weights are from an integer segment $\{1, q\}$. The probability that $C_e = 2$ is p and the probability that $C_e = 1$ is $(1 - p)$ for every $e \in E$ It is known that the m-CPSP is NP-hard problem [3].

In the [2] it is considered the case of the 2-CPSP with a common weight function for both cycles. The approximation polynomial algorithm and its approximation ratio are presented.

In the present work a more general problem 2-CPSP^d is considered: every Hamiltonian cycle has its own weight function. Nevertheless even for this more general problem the similar results were shown. Under the assumption that it is known algorithm with approximation ratio Δ for solving TSP$_{min}$(TSP$_{max}$) on the graph G, a polynomial algorithm for 2-CPSP$^d_{min}$(2-CPSP$^d_{max}$) with guarantee approximation ratio in average for all possible inputs is presented.

In particular, for the problems with edge weights 1 and 2, using an 7/6-approximation algorithm [4] for the TSP$_{min}\{1, 2\}$ the presented algorithm for the 2-CPSP$^d_{min}$ has approximation ratio $(19 - 5p)/12$, and using an 8/9-approximation algorithm [1] for the TSP$_{max}\{1, 2\}$ the presented algorithm for the 2-CPSP$^d_{max}$ has approximation ratio $(25+7p)/36$, in average for all possible inputs.

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ON THE ASYMPTOTIC OPTIMALITY OF SOLVING THE MAXIMUM-WEIGHT m-PERIPATETIC SALESMAN PROBLEM[1]

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The maximum-weight m-peripatetic salesman problem (m-PSP) can be formulated as follows. Given a complete undirected weighted graph $G = (V, E)$, the problem is to find m edge-disjoint Hamiltonian cycles $H_1, \ldots, H_m \subset E$, that maximize the sum

$$W(H_1, \ldots, H_m) = \sum_{k=1}^{m} \sum_{e \in H_k} w(e),$$

where $w : E \to R^+$ is a weight function for the edges in G. If $m = 1$, the problem is the well-known traveling salesmen problem.

The m-PSP is NP-hard in case of arbitrary weight function, as well as in case of Euclidean weight function [1]. This fact encourages development of approximation algorithms for the m-PSP with some guaranteed performance ratios.

Earlier in [2] the m-PSP$_{max}$ in a multi-dimensional Euclidean space was studied. An approximation algorithm with the cubic time complexity was constructed for this problem and the restriction on the number of salesmen m under which the algorithm is asymptotically exact was presented.

Since in most applied problems the input data are represented by real numbers with fixed number of digit positions, and taking into account the scalability of m-PSP, it is of interest to consider the special case of the Euclidean m-PSP$_{max}$ where all the vertices of graph G belong to the set of points of an integer lattice. For this problem in the present paper we construct an approximation polynomial algorithm, based on the ideas from [2] and [3]. The time complexity of the algorithm is $O(n^3)$. The conditions of asymptotic exactness of the algorithm that depend on the diameter of the input graph are obtained. For instances of the problem considered we have identified wide subclasses for which the new algorithm has better approximation ratios than the algorithm from [2].

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LOWER BOUND PROCEDURE IN THE BRANCH AND BOUND METHOD FOR THE SCHEDULING PROBLEM WITH RESTRICTED RESOURCES[1]

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We consider the resource constrained project scheduling problem with precedence and resource constraints (RCPSP). We are given a set of activities $J = \{1, ..., n\}$. The partial order on the set of activities is given by directed acyclic graph $G = (V, E)$. Each activity $j \in E$ is characterized by its processing time $p_j \in Z^+$ and the resource requirement for $r_{jk}(\tau)$ in resource type k on the time interval $[\tau - 1, \tau),\ \tau = 1, ..., p_j$. For each constrained resource type k are known its capacity R_t^k during the time interval $[t - 1, t)$. All resources are renewable. Activities preemptions are not allowed. The objective is to compute the schedule $S = \{s_j\}$ that meets all resource and precedence constraints and minimizes the makespan $C_{\max}(S)$. A formal setting of the RCPSP problem is the following:

$$C_{\max}(S) = \max_{j \in E}(s_j + p_j) \ \longrightarrow \min_{s_j} \tag{1}$$

Subject to:

$$s_i + p_i \leq s_j, \quad i \in Pred(j),\ j \in E; \tag{2}$$

$$\sum_{j \in U(t)} r_{jk}(t - s_j) \leq R_t^k, \quad k \in M,\ t = 1, ..., T_k; \tag{3}$$

$$s_j \in \mathbf{Z}^+, \quad j \in E, \tag{4}$$

where the set $Pred(j)$ is the set of immediate predecessor for activity $j \in U$, and $U(t) = \{j \mid s_j < t \leq s_j + p_j\}$ is the set of the activities which are being processed at the time instant $[t - 1, t)$ at the schedule S.

The considered problem is NP-hard. We propose to use branch and bound method to solve it. Important role in its implementation is an algorithm for computing the lower bound. As the lower bound we use the solution of the relaxed problem where all constrained resources being accumulative. To solve this problem we use a fast relaxed asymptotically exact algorithm [1], its processing times depends on the number of activities n as a function of order $n\ logn$, and the absolute error tending to zero for increasing number of activities. We present results of numerical experiments illustrating quality characteristics of the lower bound obtained by using this algorithm, the test instances for which were taken from the library of test problems PSPLIB.

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ABOUT THE BRANCH AND BOUND METHOD IN OPTIMIZATION PROBLEMS WITH INTERVAL UNCERTAINTY

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The general approach within the bounds of the branch and bound method (BBM) to solve the minimization problem in the interval formulation is substantiated in the report.

Let it be a functional $F(x)$, defined on the set X $(x \in X)$ - centered intervals; $F(x) \in X$, i.e. value that it takes, also let it be an element of the set of centered intervals. Let $D \subset X$ - is an admissible set of centered. intervals.

We introduce a linear order on the set $A = \{a_1, ..., a_k\}$ centered intervals $a_i = (\alpha_i, \sigma_i)$, $i \in J_k = \{1, 2, ..., k\}$. We introduce characteristic comparators for the centered intervals $a_i = (\alpha, \sigma)$: $(\alpha - \sigma, \alpha + \sigma) \subset R^1$, $\sigma \geq 0$:

1) $H_1 = (\alpha, \sigma) = \sqrt{\alpha^2 + \sigma^2} sign(\alpha)$.

2) $H_2 = (\alpha, \sigma) = (|\alpha| + \sigma) sign(\alpha) = \begin{cases} \alpha - \sigma, \ \alpha > 0 \\ \alpha + \sigma, \ \alpha < 0 \end{cases}$, if $H_1(\alpha, \sigma) = H_1(\beta, \delta)$. Here $sign(\alpha) = 1, alpha > 0$; $sign(\alpha) = 0, alpha = 0$; $sign(\alpha) = -1, alpha < 0$.

3) If $H_t(a_i) = H_t(a_j)$, $t = 1, 2$, that $\alpha_i \neq 0$, $H_3(a_i) = \alpha_i$, $i \in J_k$.

Such comparator we call H and denote$H =< H_1, H_2, H_3 >$.

The binary relation of order $\prec$ between intervals a_i, a_j, $i, j \in J_k$, we set as following.

1) If $H_1(a_i) < H_1(a_j)$, then $a_i \prec a_j$.

2) If $H_1(a_i) < H_1(a_j)$, $H_2(a_i) = H_2(a_j)$, then $a_i \prec a_j$.

3) If $H_1(a_i) = H_1(a_j)$, $H_2(a_i) = H_2(a_j)$, then: or a) $a_i = a_j$ $H_3(a_i) = H_3(a_j)$, say by definition: $a_i \prec a_j$ (or $a_j \prec a_i$), because $a_i = a_j$; or b) $a_i \neq a_j$, $a_i = (\alpha_i, \sigma_i)$, $a_j = (\alpha_j, \sigma_j)$ and $|\alpha_i| = \sigma_j \neq 0$, $|\sigma_i| = \alpha_j \neq 0$ (in this case $\alpha_i \neq \alpha_j$, $\sigma_i \neq \sigma_j$, $H_3(a_i) = \alpha_i$, say then $a_i \prec a_j$, if $H_3(a_i) < H_3(a_j)$; or c) $a_i \neq a_j$, $\alpha_i = \alpha_j = 0$, $\sigma_i \neq \sigma_j$, then $H_3(a_i) = \sigma_i$ and $H_3(a_j) = \sigma_j$, if $H_3(a_i) < H_3(a_j)$, that $a_i \prec a_j$.

Theorem. *The binary relation $\prec$ between the centered intervals, which is given by the comparator $H =< H_1, H_2, H_3 >$, is the linear order.*

Let $A = \{a_1, ..., a_k\}$, $a_1 \prec a_2 \prec ... \prec a_{k-1} \prec a_k$. We will call maximum a_k: $a_k = \max_{a_i \in A}\{a_i | i = 1, 2, ..., k\}$; minimum - a_1: $a_1 = \min_{a_i \in A}\{a_i | i = 1, 2, ..., k\}$.

Using the operations above centered intervals and definitions of elementary functions [1], the optimization problem on set of centered intervals D can be formulated as the following: find $\min_{x \in D} F(x)$.

In [1] BBM for minimizing of functional on the set of the intervals is proposed and justified. Further numerical experiments are necessary to establish the framework for the practical applying of the method.

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METRIC OPTIMIZATION IN SEMISUPERVISED LEARNING PROBLEMS [1]

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We consider semi-supervised learning problems in non-transductive setting, e.g. given a set of l labeled examples and $u \gg l$ unlabeled we need to predict classification of unlabeled (and probably unseen) set with high accuracy. Here we assume, that all classes contains at least one labeled object.

This problem arises in a wide variety of applications starting from medical diagnostics and image annotation to modern informational retrieval problems. One of the standard approaches to attack this problems consists of 2 stages. First, perform classification in area of high confidence. After that, algorithms classifies the rest part of the sample based on the information gathered on the first stage [4,5].

One consider within the paper SLL methods follow the schema above. Following [4] we use nearest neighbors method on the first stage. Classical obstacle here is exponential dependence of minimal number of labeled objects required for tight classification. Our contribution is generalization the method from [5] for multiclass problems. Also we reduce the bound for the minimal number of labeled examples required for discrete metric sets w.r.t. to the results proposed in [5] adapting the ideas proposed in [1–3].

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CONSTRUCTION FACTOR - SETS FOR THE ONE DIMENSIONAL PACKAGING STOCK PROBLEM [1]

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We consider the well-known one-dimensional cutting stock problem (1CSP). The one-dimensional cutting stock problem (1CSP) can be formulated as follows. Smaller pieces l_1, l_2, $\ldots$ l_m have to be cut from larger stock material L, where m is the number of items. The objective is to minimize the amount of stock material needed to produce the ordered pieces.

The instances $(L, \mathbf{l})$ and $(\tilde{L}, \tilde{\mathbf{l}})$ are proper pattern-equivalent if

$$P(L, \mathbf{l}) = P(\tilde{L}, \tilde{\mathbf{l}}),$$

where

$$P(L, \mathbf{l}) = \left\{ \mathbf{a} \ : \ \sum_{j=1}^{m} l_j a_j \leq L, \ \mathbf{a} \in \{0,1\}^m \right\}.$$

Set $P(L,\ mathbfl)$ represents a particular equivalence class. The set of all possible instances can be splitted into equivalence classes using the binary pattern-equivalence as equivalence relation.

An algorithm for enumerating all possible sets $P(L, \mathbf{l})$ for a fixed m is presented.

This method is improved for searching proper non-IRUP instances with minimal number of items. We found the minimal number of items is $m = 10$ when a proper non-IRUP instance exists. We also found 365 equivalence classes that consist of such instances.

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ON A WAVE APPROACH TO SOLVING OPTIMIZATION PROBLEMS OF LOGISTIC INFRASTRUCTURE

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The report shows two classical problems of infrastructural logistics: the problem of optimal location of service centres (a particular case of UFLP uncapacitated facility location problem [1, 2]), with segmentation of service areas, and closely associated with it the problem of optimal organizing communications (for connecting logistical centres, e.g. utilities and manufactures) [3,4]. For these problems are given mathematical statements and described algorithms of approximating solution.

At first we examine the special case of Steiner problem with specific constraints in terms of infinitely dimensional optimization. Let a bounded region $D \subseteq R^2$ has m points $A_k(x_k, y_k)$, $k = \overline{1, m}$, and piecewise continuous function $\gamma \geq f(x, y) \geq 0$ defined in D. It's necessary define shortest tree (for example in terms of minimum cost), connected points A_k

$$T(\Gamma_{i,k}) = \min_{\Gamma_{i,k}} \int\limits_{\Gamma_{i,k}} f(x, y) d\Gamma. \tag{1}$$

$$\sum_{i \in I, k \in K} T(\Gamma_{i,k}) \to \min, \tag{2}$$

where $\Gamma_{i,k} \in G_{i,k}$ – a continuous curve, connecting points A_i and A_k and giving a minimum of integral functional (1), $I \cup K = \{1, \ldots, m\}$, $G_{i,k}$ – set of various curves connecting points A_i and A_k.

To solve this problem authors worked out new algorithms, based on the wave approach, which is defined in terms of geometrical optics (Huygens principle [5,6]) and finding of global extremum of integral functional. So let one has an optical inhomogeneous medium. From a source A is beamed light waves. Each point, that is reached the front of the light wave, is secondary light source. We draw a curve of these secondary sources. This curve is the front of light wave. Thus we find the border of wave propagating. Thereby define fronts of the light wave, we can draw the extremal (moved backward at time) as a curve between the light source A and the ultimate point B. Combining this approach with the Dijkstra algorithm we obtain the desired algorithm. Also results of computation are showed.

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THE NP-HARD PROJECT MANAGEMENT PROBLEM WITH CREDITS[1]

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The project management problem with reinvestment of incomes and an opportunity to use credits is considered. There is a set of jobs $V = \{1, 2, \ldots, n\}$ and partial order of their execution E. Each job $j \in V$ is characterized by the duration p_j and the cash-flow $c_j(\tau)$, $\tau = 0, 1, \ldots, p_j$. There are financial resources $K(t)$, $t = 1, 2, \ldots, T$, where T is the planning period. The discount rate r_0 is given.

In this paper we research the model from [1], in which there is an opportunity to use credit in every moment of time. The credit rate is $r_k \geq r_0$. Let $D(t)$ be the credit size in the year t. The model with credits and reinvestment of incomes has the following form: create a schedule of jobs execution $S = (s_1, s_2, \ldots, s_n)$, where the jobs execution technological order is kept $s_i + p_i \leq s_j, \quad (i, j) \in E$, and in every moment of time $t^* = 1, 2, \ldots, T$ the positive balance of payments with credits is saved:

$$\sum_{t=1}^{t^*} \left(\frac{K(t)}{(1+r_0)^{t-1}} + \sum_{j \in N_t} \frac{c_j(t - s_j)}{(1+r_0)^{t-1}} + \frac{D(t) - (1+r)D(t-1)}{(1+r_0)^{t-1}} \right) \geq 0,$$

where $N_t = \{j \in V \mid s_j \leq t < s_j + p_j\}$ is a set of jobs which are performed in the interval $[t, t+1)$. The purpose is to maximize the net present value including credits:

$$NPV(S) = \sum_{j \in V} \sum_{\tau=0}^{p_j} \frac{c_j(\tau)}{(1+r_0)^{s_j+\tau}} + \sum_{t=1}^{T} \frac{D(t) - (1+r)D(t-1)}{(1+r_0)^{t-1}} \to \max.$$

In this paper complexity of the problem with credits was researched and the following theorem was proved.

Theorem. *The project management problem with credits is NP-hard in the strong sense.*

There is given an example that shows the necessity of credits optimization.

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AN APPROXIMATION POLYNOMIAL ALGORITHM FOR A PROBLEM OF A SEQUENCE BI-PARTIONING[1]

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We consider following strongly NP-hard [1]

Problem. *Given a sequence* $\mathcal{Y} = (y_1, \ldots, y_N)$ *of vectors from* $\mathbb{R}^q$*, and some positive integer numbers* $T_{\min}$ *and* $T_{\max}$*. Find a subset* $\mathcal{M} = \{n_1, \ldots, n_M\} \subseteq \mathcal{N} = \{1, \ldots, N\}$ *such that*

$$\sum_{j \in \mathcal{M}} \|y_j - \overline{y}(\mathcal{M})\|^2 + \sum_{i \in \mathcal{N} \setminus \mathcal{M}} \|y_i\|^2 \to \min,$$

where $\overline{y}(\mathcal{M}) = \frac{1}{|\mathcal{M}|} \sum_{i \in \mathcal{M}} y_i$*, under constraints*

$$1 \leq T_{\min} \leq n_m - n_{m-1} \leq T_{\max} \leq N, \ m = 2, \ldots, M,$$

on the elements of $\mathcal{M}$*.*

The problem is to find a partition of a finite Euclidean vectors sequence into two clusters minimizing the sum of squared distances from the clusters elements to their centers. The center of the first cluster is defined as the mean values of all vectors in a cluster. The center of the second cluster is given in advance and is equal to 0. Additionally, the partition has to satisfy the following condition: for all vectors that are in the first cluster the difference between the indices of two consequent vectors from this cluster is bounded from below and above by some constants.

In this work we present a 2-approximation efficient algorithm for the problem. This algorithm implements a dynamic programming scheme and runs in time $O(N^2(T_{\max} - T_{\min} + q))$, where $T_{\max} - T_{\min} < N$. Therefore, algorithm is polynomial and it finds the solution in $O(N^2(N + q))$ time in the general case, and in $O(qN^2)$ time in the special case when $T_{\min} = T_{\max}$.

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AN EXACT PSEUDOPOLYNOMIAL ALGORITHM FOR A TWO-CLUSTER PARTITIONING PROBLEM[1]

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In this paper we consider the following strongly NP-hard [1]

Problem. *Given a set $\mathcal{Y} = \{y_1, \ldots, y_N\}$ of vectors from $\mathbb{R}^q$ and a natural integer number M. Find a partition of $\mathcal{Y}$ into clusters $\mathcal{C}$ and $\mathcal{Y} \setminus \mathcal{C}$ such that*

$$\sum_{y \in \mathcal{C}} \|y - \overline{y}(\mathcal{C})\|^2 + \sum_{y \in \mathcal{Y} \setminus \mathcal{C}} \|y\|^2 \to \min,$$

where $\overline{y}(\mathcal{C}) = \frac{1}{|\mathcal{C}|} \sum\limits_{y \in \mathcal{C}} y$ is the center of cluster $\mathcal{C}$, under constrain $|\mathcal{C}| = M$.

In [2], a 2-approximation algorithm for the problem is proposed. The running time of the algorithm is $O(qN^2)$. In [3], a polynomial-time approximation scheme with a $O(qN^{2/\varepsilon+1}(9/\varepsilon)^{3/\varepsilon})$-time complexity, where ε is an arbitrary relative error, is substantiated. In [4], a randomized algorithm for the problem is presented. The running time of the algorithm for the fixed failure probability, relative error of the solution and for the certain value of parameter k is $\mathcal{O}(2^k q(k+N))$. The algorithm has also been proven to be asymptotically exact and to have $\mathcal{O}(qN^2)$-time complexity for the special values of the parameters.

In this work we present a pseudopolynomial algorithm which finds an optimal solution in the case of integer values of the components of the vectors in the input set and fixed space dimension. The running time of the algorithm is $\mathcal{O}(qN(2MD+1)^q)$, where D is the maximum absolute coordinate value of the vectors in the input set.

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POLYNOMIAL TIME APPROXIMATION SCHEMES FOR SOME GENERALIZATIONS OF EUCLIDEAN TRAVELING SALESMAN PROBLEM[1]

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In the Traveling Salesman Problem ("TSP"), we are given n nodes and a distance $d_{i,j}$ for each pair i, j of distinct nodes. Our aim is to find a closed path that visits each node exactly once and incurs the least cost, which is the sum of the distances along the path.

Euclidean TSP is known to be NP-hard [3]. For metric TSP N. Christofides has designed an approximation algorithm that runs in polynomial time and for every instance of problem computes a tour of cost at most $\frac{3}{2}$ times the optimum [2]. The most important result for Euclidean TSP in fixed dimensions is polynomial time approximation scheme which has been developed by S.Arora [1].

A PTAS or Polynomial Time Approximation Scheme is a polynomial time algorithm or a family of such algorithms that, for each fixed $c > 1$, can approximate the problem within a factor $(1 + \frac{1}{c})$. The running time could depend upon c, but for each fixed c there has to be polynomial in the input size.

In this paper we present generalization of polynomial time approximation scheme for Euclidean problem about two salesmen. In this problem in the plane there are n nodes and for each pair i, j of distinct nodes, there is a distance $d_{i,j}$. The goal is to construct two closed paths without common vertices that visit each node and incur the least cost.

The main idea in the algorithm of S.Arora [1] is to perform a recursive geometric partitioning of the plane with nodes. Arora proved the existence of $(1 + \frac{1}{c})$-approximate tour that crosses each line of the partition $O(c)$ times. Such a tour can be found using dynamic programming that runs in polynomial time.

In PTAS for Euclidean problem about two salesmen the parts of salesman paths have to be painted in two colors on the each step of dynamic programming to find optimal combination. It is the most important difference from Arora's algorithm. We also give estimation of running time for our generalization of polynomial time approximation scheme.

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A LOCAL SEARCH ALGORITHM FOR THE HETEROGENEOUS VEHICLE ROUTING PROBLEM

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In this work we study Heterogeneous Vehicle Routing Problems (HVRP) with limited fleet. Since it is a generalization of the classical Vehicle Routing Problem (VRP), the HVRP is NP-hard. In the classical problem, a fleet of capacitated vehicles is available to serve a set of geographically dispersed customers with known demands. Each customer should be visited by exactly one vehicle, each route should be started and finished in depot and the capacity of each vehicle should not be exceeded. The objective is to minimize the total distance traveled. In the HVRP, we use vehicles with distinct capacities, vehicle costs, and travel unit costs. The goal is to minimize the total cost of client service[3].
We developed hybrid algorithm based on Variable Neighborhood Search. It uses six well-known inter–route swap moves. Due to the specificity of the problem, these moves combine with reallocation of vehicles between routes. We also use three intra–route moves. To improve the performance of the algorithm, we use randomization of neighborhoods and auxiliary data structure. For intensification of search we use three exponential neighborhood structures based on ejection chains. For a regular change of the search area, we developed special procedure to split giant tour into routes. Algorithm was tested on well-known benchmark instances with number of customers up to 250 [1]. It showed its efficiency in comparison with other known heuristics and produced competitive results [1,2].

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APPROXIMATION ALGORITHMS FOR NP-HARD POLYHEDRAL SEPARABILITY PROBLEMS[1]

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We consider monochromatic ball covering problem as a preliminary step to solve NP-hard polyhedral separability problem.

Problem 1. Given set of points $X = \{(x_i, y_i)\}_{i=1}^{l}$ where $x_i \in \mathbb{R}^d$ and $y_i \in \{-1, 1\}$ find the smallest cardinality disjunct cover $\mathcal{D}$ of balls for X such that for every $D \in \mathcal{D}$ we have $y = 1$ for all $z = (x, y) \in X \cap D$ or $y = -1$ for each $z \in X \cap D$.

In problem 1 notations let us formulate problem 2 of polyhedral separability [2] of two subsets A and B of X having y-coordinates $y = 1$ and $y = -1$ respectively. Specifically the problem is to get the smallest cardinality set of hyperplanes such that for each pair of points $a \in A$ and $b \in B$ there is some hyperplane in the set which strictly separates a and b.

It turns out that every feasible solution $\mathcal{D}$ of cardinality N for problem 1 can be easily transformed into the feasible solution for problem 2. For $d = 2$ a power diagram (which is more general than Voronoi diagram) for disks of $\mathcal{D}$ could be obtained in time $O(N \log N)$ from which a feasible solution for problem 2 of cardinality $O(N)$ should be easily extracted in time $O(N)$. For $d > 2$ analogous transform would be done in time $O(N^2)$ with feasible solution of cardinality $O(N^2)$ to the second problem. This transform has the following statistical analogue: given clustered data find a soft power diagram having the largest margin [1].

To estimate the accuracy of the solution for problem 2 thus obtained let us denote by k_{opt} the minimal value for the problem. Also by N_{opt} we mean the smallest cardinality of the partition of X into "monochromatic" blocks (subsets whose points have equal y-coordinates) having convex hulls which contain no points of X with different y-coordinate. Then for $d = 2$ the accuracy of the solution of problem 2 could be expressed in terms of the accuracy for corresponding problem 1 as $O(k_{opt}\Delta)$ where $\Delta = N/N_{opt}$. In view of the tight bound $k_{opt} \geq \Theta(\sqrt[d]{N_{opt}})$ the accuracy would be looser for $d > 2$.

To get an approximate solution for problem 1 we use $O\left(|X|^{\lceil \frac{d}{2} \rceil} (\varepsilon^{-2} \log |X|)^{\lceil (d+1)/2 \rceil}\right)$-procedure for approximate (to accuracy $\varepsilon > 0$) ball search which contains the largest cardinality subset of A (resp. of B) while avoiding points from B (resp. from A) [3].

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LOCAL SEARCH ALGORITHM FOR THE BI–LEVEL FACILITY LOCATION AND DESIGN PROBLEM

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In the bi–level facility location and design problem, two players (a leader and a follower) compete to attract clients from a given market. Each player has a budget and tries to maximize own market share. At first, the leader opens facilities and determines their attractiveness for the clients. Later on, the follower makes the similar decision for own facilities. Each client splits own demand probabilistically over all facilities in the market proportionally with attraction to each facility and inversely proportional to the distance between client and facility. The location and attractiveness of the leader facilities are to be determined so as to maximize his market share [1].

For this Stackelberg game we present a local search heuristic based on the optimal and near optimal solutions for the follower. For a given solution of the leader, we formulate the follower problem as 0–1 nonlinear multiple–choice knapsack problem and solve it by the branch and bound method. To compute the upper bounds, we relax the integrality constraints and apply the gradient method for this convex problem. Stochastic tabu search approach is developed to find high quality feasible solution in initial node of the branching tree. Moreover, we accumulate statistical information by this local search and use it in new branching rule. The idea of backdoor branching [2] is modified to this end. Computational experiments for the test instances from [3] indicate that the local search algorithm takes a small number of steps in order to produce optimal or near optimal solutions.

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AN LP-BASED HEURISTIC FOR THE SERVERS LOAD BALANCING PROBLEM

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We consider the servers load balancing problem which is originated from optimal web hosting in cloud computing [1]. We are given a set of servers. Each server contains the set of disks (images of disks). Internet sites with heterogeneous information are distributed among the disks. A lot of users visit these sites and generate the load for the servers. The load is changed during time interval and can be characterized by some parameters: CPU, RAM and others. We assume that the activities of users are known for each site and as result for each disk and each server. If the load of the server does not exceed a given threshold, we say that the server is working in regular mode. Otherwise, it works in heavy mode. To avoid the heavy mode we can move the disks from one server to another. This moving requires some computational efforts. We will call them as overhead expenses. We assume that the expenses are known for each parameter if a disk is extracted from one server and moving to another one. Initial distribution of disks among the servers is given. The problem is to find a new distribution of the disks to minimize the total exceeding over the thresholds during the time interval subject to overhead expenses for each server.

This problem can be presented as a mixed integer linear program. Thus we can use commercial software to solve it. Unfortunately, the integrality gap is high for this formulation and the solvers can find optimal solution for small instances only. Therefore, we develop an LP-based approximate algorithm with a priori performance bound. We replace the Boolean variables by the continuous ones and solve the corresponding linear programming problem. This optimal solution allows us to fix some variables and reduces the dimension of the problem. We apply this approach until the reduced subproblem can be solved by CPLEX software. Computational results for 20 servers and 200 disks are discussed.

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APPROXIMATION ALGORITHMS FOR ENERGY EFFICIENT SCHEDULING[1]

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One of the main mechanisms used for minimizing the energy consumption in computing systems and portable devices is the so called *speed-scaling mechanism* [1], where the speed of a processor may change dynamically. If the speed of the processor is $S(t)$ at a time t then its power is $S(t)^\alpha$, where $\alpha > 1$, and the energy consumption is the power integrated over time,

$$E = \int_{t_0}^{t_1} (S(t))^\alpha dt.$$

In this setting, we consider the speed scaling scheduling problem: we are given a set $\mathcal{J} = \{J_1, \ldots, J_n\}$ of n jobs, where each job $J_j \in \mathcal{J}$ is characterized by its processing volume (work) W_j, its release date r_j and its deadline d_j, and a single (or a set of identical) speed-scalable processor(s). The processing time of a job depends on the speed. If J_j is executed at constant speed S, it takes $\frac{W_J}{S}$ time units to complete the job.

We call a schedule for the jobs in $\mathcal{J}$ *feasible* if every job is executed between its release date and its deadline. We seek for a feasible schedule of the jobs minimizing the overall energy consumption.

Most of the energy optimization scheduling problems are NP-hard. We review some of the recent methods used to derive approximation algorithms for these scheduling problems [2–4].

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GRAPH APPROXIMATION PROBLEMS (CORRELATION CLUSTERING)

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The graph approximation problems arise in analysis of systems consisting of interrelated objects, in particular, in some classification and clusterization problems. In these problems the goal is to minimize the number of links between given classes and the number of missing links within classes. The formulations and various interpretations of the graph approximation problems can be found in [1-5].

In the paper we consider only *simple* graphs, i.e., the graphs without loops and multiple edges. If $G_1 = (V, E_1)$ and $G_2 = (V, E_2)$ are labelled graphs on the same vertex set V, then the *distance* $d(G_1, G_2)$ between them is defined as follows: $d(G_1, G_2) = |E_1 \setminus E_2| + |E_2 \setminus E_1|$, i.e., $d(G_1, G_2)$ is the number of noncoinciding edges in G_1 and G_2. A simlpe graph is called an *M-graph* or a *cluster graph* if each of its connected components is the complete graph.

Graph Approximation Problem. Given an arbitrary graph $G = (V, E)$, find a nearest to G M-graph M^*.

In the different versions of the problem the number of connected components of graph M^* must be equal to a positive integer $k \geq 2$ or bounded from above by k. Moreover, the problem may involve constraints on the size of the connected components. The weighted versions of the graph approximation problems were also studied. In these problems the weight function $w : V \times V \to Z_+$ is defined and $d(G_1, G_2)$ equals to the total weight of noncoinciding edges in the graphs G_1 and G_2. In general, all versions of the graph approximation problems are NP-hard.

The graph approximation problem was formulated at first as the problem of approximating symmetric relations by equivalence relations [5]. It was intensively studied in seventies of XX century. In XXI century the graph approximation problem was repeatedly rediscovered and independently studied under different names (Correlation Clustering [3], Cluster Editing [4]).

In this survey the number of known and new results on the graph approximation problems including the results on computational complexity and performance guarantees of approximate algorithms for the graph approximation problems are presented.

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COMPUTER–AIDED WAY TO PROVE THEOREMS IN SCHEDULING MULTIPROCESSOR JOBS [1]

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We consider scheduling problems where processing of each job simultaneously requires a prespecified subset of dedicated machines. This notion is in contrast to the classical scheduling assumption that a job can only be executed on at most one machine at a time. However, a new model correctly describes execution of jobs in modern parallel computer system with shared memory. Scheduling problems with multiprocessor jobs have been considered from the middle of 80th of the last century (see [1]). For more information about these problems, we refer the reader to the survey article by Drozdowski [2].

In [3] Sevastianov and Tchernykh presented a method which allows to obtain some theoretical results for open shop problems with an aid of computer. Particularly, they obtained an upper bound on the length of the shortest schedule in three-machine open shop problem in terms of the trivial lower bound and presented 5/3-approximation algorithm.

In this paper we apply the approach by Sevastianov and Tchernykh for the scheduling problems with multiprocessor jobs. We present a branch-and-bound algorithm which allows for the problems with small number of machines to obtain the following results:
• the algorithm returns tight bounds for the optima localization intervals;
• the algorithm output solutions with provable performance guarantee;
• the algorithm distinguishes non-trivial polynomially solvable classes of instances;
• the algorithm helps to prove NP-hardness.

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EVOLUTIONARY MODELS OF DISCRETE OPTIMIZATION PROBLEMS BASED ON FRAGMENTARY STRUCTURES

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For many problems of discrete optimization the exact algorithms of polynomial complexity are not known. For these problems, it is interesting to find algorithms, which are not always accurate, but have very low complexity. Such algorithms are called "greedy algorithms". Greedy algorithms only occasionally lead to the exact optimal solution [1]. In most cases, one can only rely on the approximate solution. An interesting question is, on what classes of problems it is possible to use the greedy algorithm for finding the approximate optimal solution. For this class of problems, the solution should have a simple structure as a union of some elementary objects - fragments.

The search for classes of discrete problems, allowing the use of greedy algorithms, leads to different combinatorial structures. In particular there are matroids [2], gridoids [3], and genetic structures [4]. However, fragmentary structure is the most common configuration for similar combinatorial problems.

Definition 1. *Fragmentary structure* (X, E) *on a finite set* X *is a family of its subsets(accessible fragments)* $E = (E_1, E_2, ..., E_n)$, $E_i \subseteq X$, $i = 1, 2, .., n$, *such that* $\forall E_i \in E$, $E_i \neq \emptyset$, $\exists e \in E_i : E_i \backslash \{e\} \in E$.

Definition 2. *Fragment is called maximal, if it is not a subset of any other fragment.*

Every maximal fragment can be constructed from the empty set, successively adding elements to it, so that at each step of this procedure the resulting subset was accessible fragment. The result of applying the algorithm is specified by linear order on the set X. Thus, any maximal fragment can be described by some permutation of the elements of X.

The existence of fragmentary structure allows us to reduce a number of optimization problems to optimization problems on the set of permutations. In turn, the universal evolutionary model for optimization problems on permutations may be applied.

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BILEVEL MATHEMATICAL PROGRAMMING PROBLEMS AND TOOLS FOR DEVELOPMENT OF RAW MATERIAL BASE[1]

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The report is devoted to discussing the problem of forming a mechanism for coordinating long-term interests of the state, the private investor and the general public during the development of the raw material base on a public- private partnership (PPP). Meaningful basis approach is the concept of an state active role in the process of territory development, giving tax breaks and taking on not only a part of a general-purpose infrastructure projects , but also part of the costs of compensation for environmental losses caused by the implementation of investment projects.

The proposed research tools problem is a combination of a planning model that generates the optimal mechanism of interaction between government and the private investor, and forecasting models, the purpose of which is to produce data for the planning model and assess the effects of the area program development, using a specific mechanism of PPP.

Basic production planning model is a bilevel integer programming problem, where the state set aside the upper level and a private investor retracted lower. At the entrance of the model uses the following data:

- set of investment projects implemented by the private investor;
- set of infrastructure projects undertaken by the state;
- list of environmental projects necessary to compensate for environmental losses caused by the implementation of investment and infrastructure projects.

Model outputs are software development of mineral resource base and the PPP mechanism that identifies the costs in the implementation of infrastructure and environmental projects between the state and the investor.

Not all the original data planning problem can be obtained directly from the existing design data. Part of the data is pre-prepared in the forecasting model, significantly more detail representing the projects economy and ecology on the one hand, and especially the existing course socio-economic development on the other. Organizing iterative process at each step of the planning information base model corrects conjugate forecasting model, we obtain a complex econometric models, allowing support management decision-making process in the development of an effective mechanism for PPP.

The report is supposed to extend the core formulation including tax benefits, and also consider setting in which the top level of a multicriteria problem is solved.

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METHODS OF THE SOLUTION OF RAILWAY SCHEDULING PROBLEMS [1]

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We consider a variant of the freight railcar flow problem. In this problem, we need 1) to choose a set of transportation demands between stations in a railroad network, and 2) to fulfill these demands by appropriately routing the set of available railcars, while maximizing the total profit. We formulate this problem as a multi-commodity flow problem in a large space-time graph. Three approaches are proposed to solve the Linear Programming relaxation of this formulation: direct solution by an LP solver, a column generation approach based on the path reformulation, and a "column generation for extended formulations" approach. In the latter, the multi-commodity flow formulation is solved iteratively by dynamic generation of arc flow variables. Three approaches have been tested on a set of real-life instances provided by one of the largest freight rail transportation companies in Russia. Instances with up to 10 millions of arc flow variables were solved within minutes of computational time.

This model has a very large size, and even solving its Linear Programming (LP) relaxation using modern commercial solvers can take hours of computation time for real-life instances. However, a solution of this LP relaxation allows one to obtain a very tight dual bound for the objective function value. This fact has been also noticed for similar models considered in. Therefore, we concentrate on solving the LP relaxation of this formulation, leaving the problem of obtaining an integer solution out of the scope of the paper. In practice, rounding a fractional solution in a straightforward way allows one to obtain an integer solution with very small gap. To solve the LP relaxation faster, we devise two variants of the column generation procedure, where columns represent railcar routes or flows of the railcars of the same type. The first variant we tried is the classic Dantzig-Wolfe approach. In the second variant, called "column generation for extended formulations" in, columns are disaggregated into individual arc flow variables when added to the master. Thus, the master problem is equivalent to the original multi-commodity flow model, but its variables are generated dynamically. On almost all real-life instances provided by the company, either the first or the second variant of the column generation approach significantly outperformed the solution by a solver of the LP relaxation of the original multi-commodity model, preprocessed by a problem-specific procedure.

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OPTIMAL SCHEDULE FOR REPAIR A DOUBLE-TRACK RAILROAD [1]

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In this research it was consider the problem of the optimal scheduling of the train operation by a double track railroad when one of the segments is under repair works.

Let us assume that there exist a scheduling $\pi^{'}$ ordering all the trains running to the receiving station to pass each signal post according to the schedule. Also, there exist a set of time moments E when the segments i were closed: $E = \{[t^{'}_{i1}, t^{''}_{i1}], [t^{'}_{i2}, t^{''}_{i2}], ...\}$.

It is necessary to create a new scheduling π considering the task with the following objective function:

$$\min \sum \omega_j \max\{0, C_j(\pi) - C_j(\pi^{'})\},$$

where ω_j – is the weight of the train i, $C_j(\pi)$ and $C_j(\pi^{'})$ – are the values of the objective function for the scheduling π and $\pi^{'}$, respectively.

Application of dynamic programming is very effective for the solution of this problem. Moreover, any regular objective function can used in order to find solution of this issue.

In order to check our algorithm in the real life the test was created. We choose the part of the railway between the following stations: Costa and Babaevo which contains nine semaphores. According to the recent statistics of the Russian Railway Company approximately fifty trains ply to each direction per day. Current implementation of the algorithm can find solution to the same task with eighty trains to each direction for thirty minutes applying 8 core computer.

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APPLICATION OF SOME SWARM ALGORITHMS FOR SOLVING FACILITY LOCATION PROBLEM WITH FLEXIBLE DEMAND [1]

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The approximate methods for solution of discrete plant location problems are developed in this work. These methods are constructed on analogies with biological processes and include algorithms inspired by the "collective intelligence". These include the Ant Colony and the Bee Colony Optimization Algorithms, the Firefly Algorithm, and so on [2, 3]. Such algorithms are heuristic and, as a rule, work well from point of view of CPU time and quality of solution obtained.

In this paper, heuristics are developed for following facility location problem [1]. There is a set of demand points and a subset of potential location for plants with different work scenarios. There are pre-existing competitive facilities. Unlike the majority of facility location models the customer demand is flexible, it depends on the total utility the customer derives from this facility. In [1] this demand is called "elastic". The goal is to optimize simultaneously the locations and scenario taken into account the maximum level of budget and pre-existing competitive facilities in terms of maximizing the demand side.

In this work, a new version of Bee Colony, Ant Colony and Firefly algorithms are offered and its experimental research is carried out.

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OPTIMISATION OF BORROWER'S RISK ON MORTGAGE LOAN [1]

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When arranging a mortgage loan the borrower should think of his/her financial possibilities, as the delay of current payment leads to reconsidering bank agreement or its involuntary termination. Numerous publications about risks of mortgage loans are mostly devoted to problems the banks face at.However, the information provided for the borrowers is not enough to optimize risk on mortgage loan.

This paper investigates a model of risk minimization by a borrower when using an annuity loan scheme. A borrower having M rubles on him wants to buy a flat which costs S. Of this money a reserve Z is formed, the difference between M and Z is deposited as the first payment and the loan is received in the amount of $D = S - M + Z$ for T months.By the annuity loans monthly payments are $R = Dr\frac{(1+r)^T}{(1+r)^T-1}$, where r is a monthly loan rate. The borrower's profit at the moment T is a random quantity $\xi(t)$ with known distribution. The values of S and M are fixed and the quantities of T and Z are defined by the borrower. And D and R are dependable variables.

Let us call a borrower's risk $P(Z,T)$ as probability of shortage of money for repayment of current payment at a certain moment of time $t = 1, 2, ..., T$. Let $z(t)$ be distribution reserve at the moment t. Basically, the risk $P(Z,T)$ consider equal to zero, and $z(0) = Z$. Then we organize a cycle on t from 1 to T calculating current reserve distribution using a recurrent formula $z(t) = z(t-1) - R + \xi(t)$. Negative values of this random quantity exclude from further consideration, and $P(Z,T)$ increases by summary value of relevant probabilities. The outcome of the algorithm is the risk value depending on the loan term T and the quantity reserve Z.

There has been a program written for calculating risks and an experiment held, based on the real data as well. The calculations present that by the fixed time T the risk function has a single minimum on Z. And the risk function decreases monotonously on parameter T. But overpayment on the loan increases significantly with T rising. At the present time two-criteria model is being investigated, which takes into account both the risks and the total financial borrower's expenses.

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REINFORCEMENT SEMIDEFINITE RELAXATIONS FOR SOME NP-HARD COMBINATORIAL OPTIMIZATION PROBLEMS[1]

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Here we consider a set of popular NP-hard optimization problems, such as Max-Cut, Max-k-Cut, Correlation clustering and other (ref. [1,3]). Semidefinite programming relaxation is a popular and widely used tool to produce an effective approximation. In some cases SDP based methods gives the best possible approximation ratio (under extra hypothesis, e.g. Unique Games Conjecture [5]).

One prowed here, that using of parametrical methods [2] could strengthen classical results(in non-asymptotic case). Here we provide some theoretical guarantees, compare our method with the known ones [3,6,7,8] and give results of numerical experiments for a certain problems of Boolean circuit design.

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AN APPROXIMATION ALGORITHMS FOR THE TWO-MACHINE ROUTING OPEN SHOP PROBLEM

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We consider the two-machine routing open shop problem. It is assumed that the jobs are located at nodes of an undirected transportation network G. The machines have to travel between the vertices (with unit speed) to process the jobs. All machines use the shortest path between the nodes. The makespan of a feasible schedule is the interval between the instant when the machines start working or moving and the instant when the last machine returns to the initial node after finishing all its operations. The goal is to minimize the makespan. The considered problem is NP-hard, because it is a generalization of the two-machine open shop problem and the metric travelling salesman problem. Moreover, the problem is NP-hard even on a two-node network with two machine (see [4]).

In [1] $\frac{13}{8}$-approximation algorithm was presented for the two-machine routing open shop problem under the assumption that each machine must execute each job. We assume that each machine must perform its own set of jobs and some of jobs should be executed by both machines.

Examples of applications where machines have to travel between jobs include processing of big or heavy objects, scheduling of robots that perform daily maintenance operations, and planning of excursions in big museums $[1-3]$.

We present three approximation algorithms with the worst-case performance.

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THE BINARY CUTTING METHOD FOR SOLVING LINEAR PROGRAMMING PROBLEMS WITH BOOLEAN VARIABLES

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The report is dedicated to the description of the algorithm for one of the methods for solving Boolean variables' or mixed-integer linear programming problems, which is based on binary cuttings. The introductory part contains a brief review of discrete optimization methods development directions, of theoretical and applied aspects of discrete optimization theory. Then, the theoretical description of the main points of the binary cutting method (BCM) follows.

The author developed two BCMs for solving general integer linear programming problem, which are based on binary cuttings construction procedures. The first one uses the procedure of consecutive constructing and implicit enumeration of binary cuttings [1], while the second one is a hybrid algorithm of binary cuttings and branching, combining the idea of branch and bound method with building cutting planes. The details and the main points that underlie the binary cuttings and branching algorithm are published in [2] and are presented in the current report. It is shown that when seeking optimal solution for a linear programming problem with Boolean variables (LPPBV), it's enough to build O(n2) proper linear cuttings (PLC) for line relaxation of the initial problem. So, to create effective algorithm for solving LPPBV, it's enough to develop polynomial-difficult procedure for synthesizing PLC, which however couldn't be done to the present moment.

At the same time, we were able to develop several heuristic procedures for building binary cuttings, which allowed implementing fast algorithms to solve LPPBV. The procedures do not guarantee the construction of proper cuttings in general case, but the received a posteriori estimations of PLC's share in the total number of cuttings, and indirect estimations of overall performance, allowed making a conclusion regarding the BCM method advantage over other combinatory and hybrid methods for solving LPPBV.

The reliability of the binary cuttings and branching algorithm and its applicability for solving mixed-integer linear programming problems were experimentally confirmed.

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METAHEURISTICS FOR THE FACILITY LOCATION AND PRICING PROBLEM[1]

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We consider the bilevel mixed integer location and pricing problem which is determined by upper level and lower level problems. The upper level problem describes location and pricing process. The lower level problem simulates customer reaction on the upper level actions. First, the company opens p own facilities and defines prices on the uniform product in open facilities. After that, every customer chooses such a facility in which his total costs (product price and transportation cost) are minimal and purchases the product if his costs don't exceed his budget. The target is to find such location and pricing that the profits of the company are maximized.

It's known that this problem is NP-hard in the strong sense. We show that it belongs to the *Poly-APX*. We present two hybrid algorithms based on local search method, variable neighborhood search (VNS) and genetic algorithm. Computational experiments were conducted on instances from the "Discrete location problems" library [1]. Developed algorithms were compared with algorithms from [2] and *CPLEX* software and shown their competitiveness.

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HYBRID GENETIC ALGORITHM FOR THE OPTIMAL SYNTHESIS OF COMMUNICATION NETWORK[1]

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Consider the following problem. A simple undirected weighted graph $G = (V, E)$ with the set of nodes V, $|V| = n$, and the set of edges E is given. Let $c_{ij} \geq 0$ be *weight* of the edge $(i, j) \in E$. It is necessary to find a spanning tree $T^* \in G$ – solution of the problem:

$$\sum_{i \in V} \max_{j \in N_i(T)} c_{ij} \to \min_T,$$

where $N_i(T)$ is the set of adjacent to the vertex i nodes in the tree T.

The considered problem may be applied to the modern wireless communication networks with adjustable transmission ranges when it is necessary to minimize the total energy consumprion per time unit. In such networks the energy consumption of each element is proportional to d^s, where $s \geq 2$, and d is transmission range [1]. In this case the vertices of the graph G correspond to the network elements and the weight of an edge corresponds to the energy consumed in one time unit for data transmission between two network elements. An optimal solution of the considered problem allows to define such transmission range for each network element that the connectivity of the graph is preserved and the total energy consumption for communications is minimum. This meaningful statement of the problem is called Min-Power Symmetric Connectivity [1].

It is known that the considered problem is strong NP-hard [2,3]. We propose the genetic algorithm where the new heuristic based on variable neighborhood search is applied as mutation. The conducted numerical experiment shows the high efficiency of the proposed method.

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OPTIMAL ALLOCATION OF RESOURCES BETWEEN THE NETWORK PROJECT (BY THE EXAMPLE OF PIPELINES CONSTRUCTION)

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We consider the construction of the main pipeline in the form of a directed graph G_{ij}, whose works (i, j) are grouped by projects (sites) G_k, having different priorities of execution, where $1 \leq k \leq n$, n - number of projects. Each project has a minimum and maximum allowable probability of completion (P^* and P^{**}, respectively) in the decision-making deadline D_k project.

The basic idea of the problem of optimal allocation of resources between projects is to increase the likelihood of their completion deadlines D_k given initial investment resources company C_Σ. If any of the projects D_k in time $t \geq 0$ can not be completed in the due date with an acceptable probability, then redistributes the remaining investment resources $\sum_{k=1}^{n} C_k(t)$ between projects G_k.

As the objective function is used the sum of products of the coefficients of priority projects and their completion probabilities in the appropriate deadlines. Necessary to determine the values C_{kt}, at which the objective function is maximized:

$$\sum_{k=1}^{n} \{\eta_k \cdot P_k(C_{kt})\} \to \max,$$

following conditions:
1. $P_k^* \leq P_k(C_{kt}) \leq P_k^{**}, \quad 1 \leq k \leq n,$
2. $\sum\limits_{k=1}^{n} C_{kt} = \sum\limits_{k=1}^{n} C_k(t),$
wherein $P_k(C_{kt}) = P(t+T_k(C_{kt}) \leq D_k)$; C_{kt} – investment resources allocated by k– th project at a time $t \geq 0$, $C_{k0} = C_k$; $C_\Sigma \leq \sum\limits_{k=1}^{n} C_k$ – initial volume of investment resources for all n projects; $T_k(C_{kt})$ - random duration of the k-th project, assuming that is $P_k(C_{kt})$ linearly dependent on C_{kt}; $P_k^* = P_k(C_{kt}^*)$ - probability of completing k -th project in the due date D_k dedicated his investments C_{kt}^*; $C_k(t)$ is the remaining unused investment resources for the $t \geq 0$; η_k - priority factor (degree of importance) project.

To solve the problem, we have developed step by step algorithm, which is implemented by means of language C + + programming environment Rad Studio 2010. The algorithm is tested on the example of the allocation of financial resources for construction sites pipeline Eastern Siberia - Pacific Ocean (ESPO). Under certain restrictions, the model allows an exact solution.

BILEVEL PLANNING MODELS OF PUBLIC PRIVATE PARTNERSHIP[1]

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In this talk we introduce new bilevel models which are originated from the area of planning of public-private partnership [1].

For some models, it is shown that they are NPO-hard, and the corresponding problem of the investor is NPO-complete [2]. So, if $P \neq NP$, then for these problems do not exist a polynomial approximation algorithm with guaranteed estimate of the relative deviation from the optimum. For one of these problems we developed some algorithms based on alternating heuristic and local search method [2].

Finally, we discuss some multicriteria bilevel models and present a brief review in this area.

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DECENTRALIZED SCALABLE ALGORITHM FOR APPROXIMATE NEAREST NEIGHBOR SEARCH[1]

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The nearest neighbor search problem is an important task of many fields such as pattern recognition [1], machine learning [2], document semantic search [3].

The problem formulated as follows. Given a domain U, a metric function $\sigma : U \times U \to R$ and a finite set X. Need to preprocess X into a data structure S, so the search of nearest neighbor of q from X can be performed with a minimal numbers of metric calculation σ .

Many methods have been proposed for exact nearest neighbor search and for approximate nearest neighbor search. The search complexity of algorithms for exact nearest neighbor search has an exponential dependence from the number of dimensions. The cause lies in the "curse" of dimensionality [4]. Algorithms for an approximate version of the problem are less dependent on the number of dimensions.

In this talk we present an algorithm for approximate nearest neighbor search. We suggest to build a data structure S as graph $G(V, E)$, where $V = X$, with a small world properties and having an approximation of graph Delone as sub graph. The search algorithm based on a greedy walks and has ability to boost the accuracy without rebuilding of the structure.

Computation experiments on published datasets demonstrate performance that outperforms all existing methods.

Presented algorithms do not use the coordinate representation and do not presume the properties of Euclidean spaces, because they are based only on comparing distances between the objects and the query, and therefore are applicable to data from general metric (or even non-metric) spaces.

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THE REFINEMENT OF THE SOLUTIOIN ESTIMATION FOR AN INDIVIDUAL SET COVER PROBLEM

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The set cover problem is considered [1]. The approximation ratio of the algorithm Alg is a value $\rho(Alg)$ such that $c(Alg)/c(Opt) \leq \rho(Alg)$, where $c(Alg)$ is the upper bound for individual problems solutions that is obtained by algorithm Alg, $c(Opt)$ is the wheight of the optimal solution.

Both wheighted and non-wheighted instances of the problem are NP-hard [1]. As it shown in [2], on condition that $P \neq NP$, the greedy algorithm is the asymptotically best algorithm for the problem solution. Let Gr be a cover that is obtained by the greedy algorithm and let Opt be an optimal cover. We have [3] that $c(Gr)/c(Opt) \leq H(m) \leq \ln m+1$, where $H(m) = \sum_{k=1}^{m} 1/k$. This estimation may be refined for an individual problem: $c(Gr)/c(Opt) \leq H(m')$, where m' is the maximum cardinality of the sets in the optimal cover. Since the estimation generally cannot be improved [4], we may use some parameters of an individual problem to obtain a refinement of the estimation. Thus, for example, because it is computationally hard to obtain m', we may take the maximum cardinality of the given problem sets for m'. Also, it is possible to refine the estimaton for some special cases of the problem.

We propose a proof of the logarithmic estimation for the greedy algorithm that differs from the presented in [3] and allows us to obtain a refinement of the estimation for an individual set cover problem. For the large share of individual problems the refinement is better than the refinement $c(Gr)/c(Opt) \leq \ln m'$.

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ON INTERVAL (1,1)-COLORING OF INCIDENTORS[1]

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A proper edge coloring is called *interval* if at each vertex the set of the used colors forms an integral interval (i. e. the set of consecutive positive integers). The problem of finding an interval coloring was first considered in [1]. It was proved in [3] that this problem is NP-hard even for bipartite graphs. Note that in the case of the bipartite graphs this problem is motivated by the problem of finding a school schedule without gaps (the parts of the graph correspond to the sets of teachers and classes and an edge means that the teacher has a lesson in the class).

An *incidentor* in a graph is a pair (u, e) consisting of a vertex u an an e incident with it. So, each edge $e = uv$ has two incidentors: (u, e) and (v, e) that are called *mated* to each other. It is convenient to treat them as two halves of the edge e. Two distinct incidentors adjoining to the same vertex are called *adjacent*. An *interval* (1,1)-*coloring* of incidentors is a function from the set of incidentors into the set of colors such that all adjacent incidentors are colored by distinct colors that form an integral interval and the colors of every two mated incidentors differ exactly by 1. The main result of the paper is the following

Theorem. *If a graph has an interval edge coloring then it has an interval (1,1)-coloring of incidentors.*

It follows from this theorem that the conjecture posed in [2] that a subdivision of every interval colorable graph is also interval colorable, is true.

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THE ANALYSIS OF STABILITY OF GRADIENT ALGORITHM FOR PROBLEMS OF CONVEX DISCRETE OPTIMIZATION

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It is shown in the research that gradient algorithm is stable at curvature perturbation of admissible domain for the problems of convex discrete optimization in the terms of guaranteed estimations.

Let $Z_+^n(R_+^n)$ be the set of n-dimensional not negative integer (real) vectors, $P \subseteq Z_+^n$ – ordinal convex set [1].

It is considered the following problem A of convex discrete optimization: To find

$$\max\{f(x)|x = (x_1, \ldots, x_n) \in P \subseteq Z_p^n\},$$

where $f(x) \in \Re_p(Z_+^n)$ – not decreasing function, $\Re_p(Z_+^n)$ – class p-coordinate convex functions.

Designate curvature sets P [1] as $\theta(p)$. As usual [2], if guaranteed estimations of perturbation problem are not retrogress, then gradient algorithm is called stable.

Theorem. *Gradient algorithm is stable at "small"curvature perturbations of the set P for the problem A.*

Remark. *This research develops previously obtained results [2].*

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ABOUT A PROBLEM OF CONTROLLING A RANDOM WALK ON INTEGER POINTS PLANE [1]

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There k productions, producing two products . Each of the plants is characterized by its set of transition probabilities for the production of j, this set will denoted by $\{p_{i,j}\}, i = 1, \cdots, n,\ j = 1, \cdots, k$.. Natural to assume that each of the industries is preferable to one of the products, wherein a plurality of all walks divided into two groups by preference.

Objective is to identify in each integer point plane random walk of a given set so that minimize the likelihood of the first quadrant .

Let $(\cos\varphi, \sin\varphi)$ - food prices . For each pair walks (i, j) (one from each group) we consider the walk on the line generated by these prices. Naturally consider only agreed prices (introduced [1]) as soon as they are associated with optimum control.

Asymptotics of degeneracy for c- policy generated agreed prices and a pair of walks (i, j), is defined dimensional parameters $\lambda_{i,j}$, which can be order .

In [2] it was shown that for the agreed price angle φ must be chosen so to satisfy the equation

$$\lambda_{i,j} = \sqrt{\mu^2 + \lambda^2}.$$

Here, the pair (λ, μ) - solution associated system (see [1]) that occurs for the corresponding pair of walks.

Thereby , a plurality of pairs should be chosen such that $\sqrt{\mu^2 + \lambda^2}$ is maximal.

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EQUIVALENT TRANSFORMATIONS IN ORTHOGONAL PACKING PROBLEM[1]

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We consider Minimum Raster Set Problem (MRSP) that can be formulated in the following way. Let we have the tuple $(L, \mathbf{l}) \in R_+ \times R_+^m$, where $0 < l_1 \leq l_2 \leq \cdots \leq l_m \leq L$. Define the raster set as

$$R(L, \mathbf{l}) = \left\{ \sum_{j=1}^{m} l_j a_j \ : \ \sum_{j=1}^{m} l_j a_j \leq L, \ \mathbf{a} \ \in \{0,1\}^m \right\}.$$

Two tuples $(L, \mathbf{l})$ and $(\tilde{L}, \tilde{\mathbf{l}})$ are equivalent if:

$$P(L, \mathbf{l}) = P(\tilde{L}, \tilde{\mathbf{l}})$$

where

$$P(L, \mathbf{l}) = \left\{ \mathbf{a} \ : \ \sum_{j=1}^{m} l_j a_j \leq L, \ \mathbf{a} \in \{0,1\}^m \right\}.$$

We need to find the tuple $(L^*, \mathbf{l}^*)$ equivalent to $(L, \mathbf{l})$ with minimum number of raster points $|R(L^*, \mathbf{l}^*)|$.

This problem arises in representation of defferent problems of discrete optimization as integer linear programs where number of variables depends on number of raster points (Arc-flow models [1,2,3]).

We propose a method for finding bounds for MRSP based on linear programming. We consider different ways of constructing the equivalent tuple $(L^*, \mathbf{l}^*)$ with different objective functions to get the upper bound. We also propose a method for estimating the lower bound. Computational results are presented as well.

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A FEW ALGORITHMS FOR SOME QUADRATIC EUCLIDEAN PROBLEMS OF CHOOSING VECTOR SUBSET AND SUBSEQUENCE [1]

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We consider following strongly NP-hard [1]

Problem. *Given a sequence* $\mathcal{Y} = (y_1, \ldots, y_N)$ *of vectors from* $\mathbb{R}^q$*, and positive integer numbers* $M > 1$*,* $T_{\min}$ *and* $T_{\max}$*. Find a subset* $\mathcal{M} = \{n_1, \ldots, n_M\} \subseteq \{1, \ldots, N\}$ *such that*

$$\sum_{n \in \mathcal{M}} \|y_n - \overline{y}(\mathcal{M})\|^2 \to \min,$$

where $\overline{y}(\mathcal{M}) = \frac{1}{|\mathcal{M}|} \sum_{i \in \mathcal{M}} y_i$*, under constraints*

$$1 \le T_{\min} \le n_m - n_{m-1} \le T_{\max} \le N, \ m = 2, \ldots, M,$$

on the elements of $\mathcal{M}$*.*

In the case of $T_{\min} = 1$ and $T_{\max} = N$ the formulated problem is equivalent to one strongly NP-hard problem [2] of choosing «compact» subset in $\mathcal{Y}$.

In this work we present several algorithms for considered problems.

For the problem of choosing a vector subset we have proposed:

– 2-approximation algorithm having a $\mathcal{O}(qN^2)$-time complexity;

– an exact pseudo-polynomial algorithm with $\mathcal{O}(qN(2MB)^q)$-time complexity for the case when the dimension q of space is fixed and the vectors have integer components, here B is maximal absolute value of input vectors components;

– fully polynomial time approximation scheme (FPTAS) for the case when the dimension q of space is fixed; the presented algorithm builds a $(1+\varepsilon)$-approximate solution in $\mathcal{O}(N^2(M/\varepsilon)^q)$-time.

For the problem of choosing a vector subsequence we have proposed:

– 2-approximation algorithm with $\mathcal{O}(N^2(q + N^2))$-time complexity;

– an exact pseudo-polynomial algorithm with $\mathcal{O}(N(q + N^2)(2MB)^q)$–time complexity for the case when the dimension q of space is fixed and the vectors have integer components.

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ALGORITHMS FOR SOME BATCHING SCHEDULING PROBLEMS WITH BOUNDED SIZE OF BATCHES [1]

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We consider the following scheduling problem. The customer i places n_i orders to the manufacturer. Order O_{ij} requires a processing time p_{ij} and its due date is d_{ij}, $j = 1, \ldots, n_i$, $i = 1, \ldots, s$. The manufacturer produces the orders sequentially. For delivery to the customer orders are distributed to the batches. Time completion of the order is the time completion of the batch to which this order is placed. The number of batches is bounded by k, the number of orders in each batch does not exceed m (assuming $mk \geq n$).

In [3] for one customer case we prove polynomial solvability of maximum tardiness minimization problem ($T_{\max}$) and total completion time minimization problem (C_Σ). Also we show NP-hardness of total weighted completion time minimization problem ($C_{\Sigma w}$).

In this paper we propose the algorithm for finding an approximate solution and some polynomially solvable cases of total weighted completion time minimization problem in the case of one customer. Also we propose algorithms for three formulations of maximum tardiness minimization problem with several customers. These algorithms show good results during the computational experiment.

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THE POLYTOPE OF THE GRAPH APPROXIMATION PROBLEM

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In this paper we consider the following problem. Let $K_n = (V, E)$ is a complete graph on n vertices without loops and multiple edges, G - subgraph of K_n. Denote by $\mathcal{M}(V)$ the family of all subgraphs of K_n, which connected components are cliques.The graph approximation problem is to find a graph $M = \mathcal{M}(V)$ that minimizes the functional $\rho(G, M) = |EG \cup EM| - |EG \cap EM|$.

This problem is NP-hard In the general case [1]. For this problem, some known polynomially solvable cases [2], constructed estimates of the objective function [3], developed approximate algorithms [4].

We consider the polyhedral structure of the graph approximation problem. The vector incidence of subgraph $H \subset K_n$ is a vector $x^H \in R^E$, where R^E - space associated with the set E, with coordinates $x_e^H = 1$ if $e \in EH$ and $x_e^H = 0$ if $e \notin EH$. Correspondingly, the polytope of this problem is a set $P = conv\{x^M \in R^E | M \in \mathcal{M}(V)\}$.

Theorem 1. *Polytope P is the convex hull of integer solutions of the system*

$$\begin{aligned} x_{uv} + x_{u\omega} - x_{v\omega} &\leq 1 \\ x_{uv} - x_{u\omega} + x_{v\omega} &\leq 1 \\ -x_{uv} + x_{u\omega} + x_{v\omega} &\leq 1, \\ x_{uv} &\geq 0, \end{aligned} \tag{1}$$

where $u, v, \omega \in V$ are possible triples of distinct vertices.

In these terms, the objective function $\rho(G, M)$ for a given G will have the form

$$f(x) = |EG| + \sum_{e \in E\bar{G}} x_e - \sum_{e \in EG} x_e.$$

Theorem 2. *Each constraint of the system (1) defines a facet of the polytope P.*

In addition, the new classes of facet inequalities of P are describes in this paper. For some classes are solving the separation problem.

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HEURISTICS FOR MINIMUM SPANNING K-TREE PROBLEM[1]

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We consider a NP-hard combinatorial optimization problem of finding a spanning k-tree [1] of minimum weight in a complete weighted graph, known in literature as *Minimum Spanning k-Tree Problem* (MSkT). MSkT has a number of applications in designing reliable telecommunication networks [2] and generalizes a classical problem in graphs, the *Minimum Spanning Tree Problem* [3].

Definition. [1] *A k-tree is a member of a class of undirected graphs defined recursively as follows: complete graph with k vertices is a k-tree; if T is a k-tree with n vertices, then a new k-tree with $n+1$ vertices is formed by creating a new vertex v and adding edges between v and every vertex of an existing k-clique in T.*

The mathematical formulation of the MSkT is as follows. Let $G = (V, E)$ be a complete weighted undirected graph, where V is a set of nodes and E is the set of edges, and for each edge $[i, j] \in E$ the weight $w(i, j) \geq 0$ is given. Let $T(G)$ be a set of all spanning k-trees in a graph G, where a spanning k-tree is a k-tree that contains all the vertices and a subset of the edges of a graph G. Let $w(T)$ be a weight of edges of the spanning k-tree $T \in T(G)$. It is required to find a spanning k-tree T^* of minimum weight in a complete weighted graph G: $T^* = \arg\min_{T \in T(G)}\{w(T)\}$.

We propose effective heuristics are based on the idea of a well-known Prim's algorithm and based on a dynamic programming approach. We also propose metaheuristics: ant colony algorithm, variable neighborhood search algorithm and genetic algorithm. Preliminary numerical experiment was performed to compare the effectiveness of the proposed algorithms with known heuristics and exact algorithms.

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POLYHEDRAL PROPERTIES OF THE PROBLEM OF INTERRUPT-ORIENTED SERVICES OF JOBS BY SINGLE MACHINE

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A problem of services of various jobs by single machine is contemplated here [1]. There is a set of jobs defined $V = \{1, 2, \ldots, n\}$. Each job $i \in V$ has positive weight ω_i, processing time p_i and release date r_i, during which it is unavailable for services. Processing time for various jobs are equal to each other $p_i = p$. In the process of work of machine interruptions are assumed. It is necessary to minimize total weighted completion time of serving all the jobs. Suppose $D = \{1, \ldots, d\}$ is instant of time set, sufficient for serving all the jobs.

Here we contemplate the model of integer linear programming of the problem herein [2].

It is required to minimize the function $g(x, y) = \sum\limits_{i \in V} \omega_i + \sum\limits_{i \in V} \omega_i (\sum\limits_{k=1}^{d-1} y_{ik})$ under condition that

$$\sum_{i \in V} x_{ik} \leq 1, \ k = 1, \ldots, d; \quad \sum_{k=1}^{d} x_{ik} = p, \ i \in V; \tag{1}$$

$$\frac{1}{p} \sum_{l=k+1}^{d} x_{il} \leq y_{ik} \leq \sum_{l=k+1}^{d} x_{il}, \ i \in V, \ k = 1, \ldots, d-1; \tag{2}$$

$$x_{ik} \geq 0, i = 1, \ldots, n, \ k = 1, \ldots, d; \tag{3}$$

$$x_{ik} = 0, i = 1, \ldots, n, \ k = 1, \ldots, r_i; \tag{4}$$

$$y_{ik} \leq 1, i \in V, \ k = 1, \ldots, d; \tag{5}$$

where variables are set as the following

$$x_{ik} = \begin{cases} 1, & \text{if } i \in V \text{ in the instant of time } k \in D, \\ 0, & \text{otherwise;} \end{cases}$$

$$y_{ik} = \begin{cases} 1, & \text{if by the instant of time } k-1 \text{ the job } i \text{ is not completely served yet,} \\ 0, & \text{otherwise.} \end{cases}$$

In the work the properties of polyhedron (1) - (5) for small p are contemplated.

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SOME APPROXIMATE ALGORITHMS FOR NP-COMPLETE PROBLEM OF SEQUENCING

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For strongly NP-complete scheduling problem - minimizing maximum lateness some properties of optimal permutations is proved and on their basis approximate algorithms for the solution of the problem is developed.

n jobs have to be processed on a single machine not earlier than the time t. Machine can process at most one job at a time and preemption in the process of any job is not allowed. Assume that the jobs are specified numbers from 1 to n. Assume $N = \{1, 2, ..., n\}$. The following data spesified for each job j, $j \in N$: a release date r_j; a processing time $p_j \geq 0$; a due date d_j. Numbers t, r_j, p_j, d_j are integer. We will understand a permutation of all elements the set of jobs N as the schedule. We will denote $\Pi(N, t)$ the set of schedules on the set of jobs N which are begining at time t. The problem consists in searching of such schedule π^* that $\max\limits_{j \in N}\{t_j(\pi^*) - d_j\} = \min\limits_{\pi \in \Pi(N,t)} \max\limits_{j \in N}\{t_j(\pi) - d_l\}$, where $t_j(\pi)$ is completion time job j in the schedule π. Schedule π^* is called an optimal.

Let $N' \subseteq N$, $N' \neq \oslash$, $t' \geq t$, the j_d is job with smallest due date from N', $\pi \in \Pi(N', t')$.

Assume $J_{max}(\pi, j_d) = \{j \in N' \setminus \{j_d\} : r_j \geq r_{j_d}, j_d \xrightarrow{\pi} j\}$, $J_{min}(\pi, j_d) = \{j \in N' \setminus \{j_d\} : r_j < r_{j_d}, j_d \xrightarrow{\pi} j\}$. We note that the sets $J_{max}(\pi, j_d)$, $J_{min}(\pi, j_d)$ can be empty. In paper [1] existenceis optimal schedule π^* that $J_{max}(\pi^*, j_d) = \{j \in N' \setminus \{j_d\} : r_j \geq r_{j_d}\}$ is proved and a general scheme for finding this schedule is constructed assuming that the set $J_{min}(\pi^*, j_d)$ can be founded some algorithm A complexity $O(x(n))$, where $x(n)$ is function which dependent from dimension of the problem.

Scheme [1]. Let assume $t_1 = \max\{r_{min}(N), t\}$, $N_1 = N$, $\pi_1 = \pi^{\oslash}$. Let known t_k, N_k, π_k, и $k \geq 1$. If $N_k = \oslash$ then π_k is optimal schedule and process is complete. Otherwise, we choose the job j_d^k with smallest due date from N_k, $J_{max}^k = \{j \in N_k \setminus \{j_d^k\} : r_j \geq r_{j_d^k}\}$, some algorithm A is finding the set $J_{min}^k = J_{min}(\pi^*, j_d^k)$ and assume $N_{k+1} = J_{min}^k \cup J_{max}^k$, $N^k = N_k \setminus (J_{max}^k \cup J_{min}^k \cup \{j_d^k\})$, $\pi_{k+1} = (\pi_k, \vec{\pi}_r^k, j_d^k)$, where $\vec{\pi}_r^k \in \vec{\Pi}_r(N^k, t_k)$, $t_{k+1} = T(\pi_{k+1})$.

Some properties of optimal schedules are proved and on their basis pseudopolynomial algorithms for finding sets J_{min}^k in the realization of the scheme are developed. The number of iterations of algorithms less than n. At each iteration of the first algorithm the job with a biggest release date from not sequence jobs is add to schedule which constructed on the previous step on the place determine as special way. At each iteration of the second algorithm the job with a smalest release date from not sequence jobs is sequenced, structure schedules which constructed on the previous step is partially saved .

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ABOUT RESEARCH AND SOLVING THREE-DIMENSIONAL EXTENDED VEHICLE ROUTING PROBLEM[1]

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Three-Dimensional Extended Vehicle Routing Problem(3L-EVRP) includes solving of next sub problems: demand prediction of the desired product; calculating rational routes for product delivery by lorries with different capacity (taking into account time windows, planing period, split delivery, multi depot, inhomogeneity of carport) [1]; choosing of items for transportation; packing cylinders and parallelepiped items into containers taking into account weights and palletisation, which should be less then capacity of lorry [2]. Algorithm P-ACO-3L-EVRP on the base of metaheuristic ant colony based on population taking into account some restrictions, was developed for searching rational routes. Evolutionary algorithm $(\lambda + \mu)$ EA3D was developed for solving packing problem. Algorithms, developed for solving problems mentioned above, was combined into one unit, which could be called prototype of transportation logistical system.[2]. Estimation of algorithm efficiency for P-ACO-3L-EVRP и $(\lambda + \mu)$EA3D was obtained by computing on the test cases of OR-library. Time windows, planing period, split delivery, multi depot, inhomogeneity of carport takes into account for vehicle routing problem. Results compared with genetic algorithm (G.Jeon, H.R. Leep, J.Y. Shim), TABU search algorithm (C. Archetti, M.G. Speranza, A. Hertz). Best value of objective function was obtained for four test cases. $(\lambda + \mu)$ EA3D algorithm checked on well known test cases of OR-library, and compared with genetic algorithm(Bischoff, Ratcliff), several kinds of hybrid metaheuristic algorithm (Liang, Lee, Liang). Algorithm shows commensurate results for the best known.

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ON AN APPROACH TO FINDING ROBUST SOLUTIONS TO DISCRETE LOCATION PROBLEMS [1]

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We consider two fundamental problems in discrete location theory — the uncapacitated facility location problem and the p-median problem, that can be formulated in the following. Given a set $J = \{1, \ldots, n\}$ of clients, a set $I = \{1, \ldots, m\}$ of potential sites, costs d_{ij} of satisfying the demand of client $j \in J$ from the facility located at site $i \in I$, and the demand ω_j of client $j \in J$. The p-median problem is to choose exactly p sites for opening facilities in order to minimize the overall total cost of satisfying the demands of all clients

$$\min_{S \subseteq I} \Big\{ \sum_{j \in J} \omega_j \min_{i \in S} d_{ij} : \ |S| = p \Big\}.$$

In the uncapacitated facility location problem the number of facilities to be opened is not fixed in advance. Instead, there are fixed costs f_i of locating a facility at site i. The problem consists in minimizing both the total transportation cost and a cost associated with the plants to be opened.

For two discrete location problems presented above we consider robust versions [3] based on a threshold model, which concept was introduced in [1, 2]. The main idea of the approach is that the demand of each client may be unknown in practice precisely, but some estimates $\widehat{\omega}_j$ are given instead. In the threshold model, a threshold value $\tau > 0$ is introduced, to be interpreted as the highest admissible total cost (or budget) of satisfying the demands of all clients. In this case the robustness $\rho(S)$ of a solution $S \in I$ is defined as the smallest difference between the demands ω_j and their estimates $\widehat{\omega}_j$, which makes the total cost exceed the threshold τ. In the case of the p-median problem the robustness of a solution $S \in I$ can be written as follows

$$\rho(S) = \min \Big\{ \|(\omega_j)_{j \in J} - (\widehat{\omega}_j)_{j \in J}\| : \sum_{j \in J} \omega_j \min_{i \in S} d_{ij} > \tau \Big\}, S \subseteq I, |S| = p,$$

In the talk we consider bi-criteria versions of the discrete location problems presented above with one additional criterion maximizing the robustness. A method of finding Pareto optimal solutions based on a well-known approach in the field of bi-criteria optimization is proposed and the results of the extensive computational experience are demonstrated.

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PACKING METHODS OF N-DIMENSIONAL ORTHOGONAL POLYHEDRAL

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The report considers the problem of packing n-dimensional items defined structure in closed bounded n-dimensional object. It is known fact that cutting-packing problems belong to the class NP-hard problems.

N-dimensional orthogonal polyhedral (n-OP) is an object consisting of a finite number of disjoint n-dimensional rectangular parallelepipeds. An important point in solving the general Packing Problem is designing method of packing (pattern) (see [1]).

The report proposes an algorithm of coordinate-wise stacking, consisting of two stages. On the ground there is preparation n-OP to his placement: each n-OP compared sequences of rectangular blocks (tuple-functions). On the second - finding the coordinates of the initial point of the n-OP in packing object with the necessary and sufficient conditions for the admissibility of the packing (see [2]).

The report proposed a method for obtaining packing lower boundary based on dual fusible functions. The numerical experiment held.

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SOLVING OF MINIMAX WEBER PROBLEM IN A PLANE WITH FORBIDDEN GAPS[1]

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Minimax Weber problem is defined as follows. The problem is to locate n new facilities $X_1, ..., X_n$ in the plane when there are m facilities $P_1, ..., P_m$ already located. There are the forbidden rectangular gaps F_k, which sides are parallel to axes of coordinates, $F = \bigcup F_k$, $k = 1, ..., z$; $w_{ij} \geq 0$ and $v_{jk} \geq 0$ — costs of communications between P_i and X_j, X_j and X_k respectively. We need to locate facilities $X_1, ..., X_n$ out of the forbidden gaps so as to minimize the most weighed distance between all facilities [1, 2]. The mathematical model is:

$$\max\{\max_{1\leq i\leq m, 1\leq j\leq n} w_{ij} d(P_i, X_j), \max_{1\leq j<k\leq n} v_{jk} d(X_j, X_k)\} \to \min, \tag{1}$$

$$X_j \notin Int\ F,\ j = 1, ..., n, \tag{2}$$

where $d(\cdot, \cdot)$ — some metric, $Int\ F$ — interior of F.

Results of researches of the problem without restrictions (2) for the rectangular metric are given in [3]. In [1] models of integer linear programming for minimax and minisum Weber problem with the forbidden gaps are constructed. A variant of algorithm of branch and bounds is proposed in work [2].

In the report problem (1)–(2) with rectangular metric is considered. It is proved that for search of the optimum sufficient to consider a subset of admissible solutions. Some variants of the bottom estimates of criterion function are proposed. A modification of the algorithm [2] is developed. Results of the computing experiment in comparison of efficiency of the algorithm and the solving of the problem by means of integer linear programming model and package IBM ILOG CPLEX is represented.

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EXPERIMENTAL ANALYSIS OF SOME CLASS OF SET PACKING PROBLEMS[1]

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The well-known NP-hard set packing problem (SPP) is considered in the integer formulation:

$$\max\{cx \mid Ax \leq e,\ x \in \{0,\ 1\}^n\}.$$

Here $A = (a_{ij})_{m\times n}$ is boolean matrix, c is an n-dimensional positive vector, $e = (1, ..., 1)^T$ is m-dimensional vector and $x = (x_1, ..., x_n)^T$ is the vector of the variables. We study the class $\mathcal{P}_{n,p}$ of the SPPs, where all the elements of the matrix A are independent random variables, $\mathbf{P}\{a_{ij} = 1\} = p$, $\mathbf{P}\{a_{ij} = 0\} = 1 - p$, where $p \in (0,\ 1)$, $i = 1, ..., m$, $j = 1, ..., n$. Earlier in [1, 2] for certain subclasses of $\mathcal{P}_{n,p}$ we obtained polynomial upper bounds of the average number of iterations for some algorithms of solving of SPP based on continuous optimization techniques.

The report presents the results of experimental studies of the properties of problems of class $\mathcal{P}_{n,p}$, including duality gap, cardinality L-covering and the number of feasible solutions. Also we analyze the process of solving this problem by some cutting plane algorithms, the branch-and-bound method (the Land and Doig scheme) and L-class enumeration algorithm. In particular, the comparison of the fractional Gomory cutting plane algorithm and of its modification [1] showed the advantage of the modified version, which increases with extension of the interval for the coefficients of the objective function. The results obtained will be used to improve the efficiency of algorithms for solving the set packing problem.

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METHODS OF APPROXIMATE SOLUTION SEARCHING FOR A SHORTEST PATH PROBLEM IN A LARGE GRAPH

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As computer technology advances, modeling and solving problems based on large data sets (Big Data) becomes increasingly more relevant. Since the beginning of the 16st century, problems relating to the processing of large data sets have started to appear in many different areas. For example: astronomical observations and calculations of the planets' trajectories; weather forecasting and natural disaster prediction; data processing of trading operations; and searching optimal sea routes.

Much of the research which is conducted using big data analysis can be modeled using large graphs. Examples of the large graphs include: web graph (mathematical model of the Internet); Gmail e-mail messages graph; European road network; FriendsGraph on Facebook; Google Knowledge Graph; and citation graph (graph based on citations of academic publications).The number of vertices and edges in such graphs is estimated to be millions and billions. Usual algorithms, including shortest path algorithms – like Dijkstra and Floyd-Warshall – are ineffective and require a lot of memory to store the graph's intermediate states.

This Report presents modern approaches to the construction, processing and storage of large graphs, adopted by big companies such as Google and Microsoft. A fairly popular approach is to use two-phase algorithms ALT (Goldberg, Harrelson, 2005), TN (Geisberger, 2008), HL (Abraham, 2011). The first phase includes graph pre-processing and saving the results to a database. The second phase includes database query execution and final calculations. A basic assumption is that a road network varies only slightly over time, so it's not necessary to frequently conduct the processing phase.

Another area where it is important to be able to correctly handle large graphs is where there are problems of effective flow management in the transportation system; searching for optimal routes; making decisions about road network development; and as a network model typically used as a road graph. The process of building the road graph based on actual data about the current state of the network is described in a paper [1]. However, in the problems described above, the network changes frequently and therefore two-phase algorithms are not applicable. In this case, the bidirectional Dijkstra's algorithm was selected, with a preliminary stage of the road network transformation to a smaller size subnet.

This paper presents a comparative analysis of two-phase algorithms ALT and TN on Russia's road network and develops recommendations for using various shortest path algorithms in different cases.

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CONSTRUCTION OF A MINIMAL COVERING OF INCLUSION DEPENDENCES

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In paper automatic construction of an irredundant set of relationships on the scheme of a database which in the literature usually refer to as foreign keys [1] is considered.

The formal base for an relationships forming are inclusion dependences.

Definition 1. *Suppose that $R_i[A_1, ..., A_m]$ and $R_j[B_1, ..., B_p]$ — are relations of database (it is possible that they equals). $V \subseteq \{A_1, ..., A_m\}$ and $W \subseteq \{B_1, ..., B_p\}$, where $|V| = |W|$, and condition $\pi_V(R_i) \subseteq \pi_W(R_j)$ is held, then $R_i[V] \subseteq R_j[W]$ object is called inclusion dependence.*

Where $|V|$ – cardinal number of V, $\pi_V(R_i)$ – projection of relation R_i on attributes V.

Let's turn to definition of relations between tables in databases, which are often used as reference integrity constraint.

Definition 2. *The relationship $L_1(i, j, X)$ is existed between relations R_i and R_j if the condition $PK(R_i) = PK(R_j)$ has been satisfied, and condition $\pi_X(R_j) \subseteq \pi_X(R_i)$ has been satisfied too for any of R_i u R_j where $X = R_i \cap R_j$, and $PK(R_i)$ – a primary key of ration R_i.*

Definition 3. *The relationship $L_M(i, j, X)$ is existed between tables R_i and R_j if the conditions $PK(R_i) \neq PK(R_j)$ and $PK(R_i) \subseteq R_j$ have been satisfied.*

Using properties of a minimal covering of functional dependences [2, 3], we shall receive following properties of inclusion dependences.

Theorem. *The relationship $L(i, j, X)$ is redundant, if following relationships are existed:*

$$L(i, m(1), X_0),\ L(m(1), m(2), X_1), ..., L(m(p), j, X_p), \tag{1}$$

and $X \subseteq PK(i) \subseteq X_s \subseteq R_{m(s)},\ s = 2, 3, ..., p$.

The algorithm of formation of sequences (1) is a polynomial of the second order with respect to the total number of inclusion dependencies.

Algorithms of automatically generate of referential integrity constraints was implemented in accordance mathematical statementsmentioned above and accordance to known mechanism of using relationshipson the databasescheme. The correctness and irredundant of theconstructionresults was proved. The software that implements the automatic construction of a complete irredundant set of relationshipson database scheme was developed.

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CONTINUOUS OPTIMIZATION

THE SOLVER MODULE FOR LINEAR STOCHASTIC PROBLEMS[1]

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The work is devoted to creation of software for the solution of linear stochastic problem of the type:

$$M\left(\sum_{j=1}^{n} c_j x_j\right) \to max,$$
$$P\left\{\sum_{j=1}^{n} a_{ij} x_j \le b_i\right\} \ge \alpha_i, i = 1, ..., m, \tag{1}$$
$$x_j \ge 0, j = 1, ..., n.$$

There are implemented two approaches for resolve problem (1).

The first approach – move to deterministic task.

It is known [1], if the elements of the matrix A and components of the vector b are mutually independent normally distributed random variable $a_{ij} \in N(\overline{a}_{ij},\ \sigma_{ij}^2)$, $b_i \in N(\overline{b}_i, \theta_i^2)$ and the condition $\alpha_i \ge 0.5,\ i = 1, ..., m$, then the problem (1) is reduced to deterministic problem of convex programming in the following form:

$$\sum_{j=1}^{n} \overline{c}_j x_j \to max,$$
$$\Phi^{-1}(\alpha_i)\left\{\sum_{j=1}^{n} \sigma_{ij}^2 x_j^2 + \theta_i^2\right\}^{\frac{1}{2}} + \sum_{j=1}^{n} \overline{a}_{ij} x_j \le \overline{b}_i, i = 1, ..., m, \tag{2}$$
$$x_j \ge 0, j = 1, ..., n.$$

For the solution of problem (2), provided that $x \in X$, where X – convex set, in the software package implements a method possible directions. In addition, there was conducted study based on statistical methods and simulation [2], the result of which are the conditions, in witch possible to use problem (2) to find the solution of problem (1) if the elements of the matrix A and vector b are mutually independent uniformly distributed random variable $a_{ij} \in R(\ \underline{a}_{ij},\ \overline{a}_{ij})$, $b_i \in R(\underline{b}_i,\ \overline{b}_i)$.

The second approach – a direct method for solving stochastic problems.

In developed software implemented design method of stochastic quasigradient [3] for solving problem (1), provided that $x \in X$, where X – convex set.

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AN APPROACH TO GLOBAL OPTIMIZATION BASED ON MULTIDIMENSIONAL ADAPTIVE GRIDS[1]

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The talk is devoted to methods for generation of adaptive interpolation grids for multidimensional numerical optimization. We study extremal problems of the form: Minimize $f(x)$ over the set

$$D = \{x \in \mathbb{R}^n : \underline{x} < x < \overline{x}\}.$$

Here, $\underline{x}, \overline{x} \in \mathbb{R}^n$ are given vectors. Under the lack of a priori information about analytical properties of the function f, we meet a problem with a so-called "black-box" cost function [2]. In numerical algorithms for such global optimization problems, computing time (i.e. an average calculation interval of a value of the function) is customary high, and plays a crucial role.

Adaptive grids is a popular issue in numerical mathematics (see, e.g., [1]). The feature of such grids is their irregularity: Nodes of a grid are crowded in neighborhoods of critical points of a cost function, while in the remainder of D, the grid can be left rather sparse. As a rule, this leads to a tangible raise of algorithms' efficiency.

The proposed approach employs a modified Sheppard function [3] as a reference function. Introduce a set $\mathcal{D} = \{x_k : \ x_k \in D, k = \overline{1, N_{\mathcal{D}}}\}$ of randomly chosen points. The modified Sheppard function $F_S(\cdot; \mathcal{D})$ is defined as follows [3]: On $\mathcal{D}$ it coincides with f, and on $D \setminus \mathcal{D}$ it is given by the formula

$$F_S(x; \mathcal{D}) = \sum_{k=1}^{N_{\mathcal{D}}} \frac{f_k + a_k(x - x_k)}{\left\|x - x_k\right\|^4} \Big/ \sum_{k=1}^{N_{\mathcal{D}}} \frac{1}{\left\|x - x_k\right\|^4}.$$

Here, $f_k = f(x_k)$, and $a_k \in \mathbb{R}$ are coefficients of inclination angles of the function F_S at points $x_k \in \mathcal{D}$.

In the talk we discuss results of numerical implementation of problems from collection [4]. The proposed approach makes sense in ravine unimodal, and multiextremal problems.

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AVERAGE COST MINIMIZATION PROBLEM

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It is well known that short run cost functions of firms are convex functions when production functions are concave[3]. Average cost minimization problems are often considered in the literature[1-3]. However, it seems that less attention so far has been paid to properties of average cost functions and numerical methods for solving the average cost minimization problem. Aim of this talk is to fulfill this gap. First, we show that average cost functions are pseudoconvex. Second, we develop an algorithm for solving the average cost minimization problem. We implemented the algorithm in the case of some companies of Mongolia.

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SEPARABILITY PROBLEMS OF TWO SETS BY PIECEWISE LINEAR AND QUADRATIC FUNCTIONS AS OPTIMIZATION PROBLEMS[1]

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The separability problem of the two finite sets arising in classification problems, formulated as follows [1]. Let there be given two finite sets $\mathcal{A} = \{a^1, \ldots, a^M\}$ and $\mathcal{B} = \{b^1, \ldots, b^N\}$ in the space $I\!R^n$. The separability problem of the sets $\mathcal{A}$ and $\mathcal{B}$ consists in finding a discriminant function $F(\cdot)$ such that $F(x) > 0 \ \forall x \in \mathcal{A}, \quad F(x) < 0 \ \forall x \in \mathcal{B}$.

The classical problem of linear separation [2] consists in constructing a hyperplane $H(\omega, \xi) = \{x \in I\!R^n | \ \langle \omega, x \rangle = \xi\}$ such that $\langle \omega, x \rangle > \xi \ \forall x \in \mathcal{A}, \ \langle \omega, x \rangle < \xi \ \forall x \in \mathcal{B}$. A linear separation problem is equivalent to a linear programming problem. Unfortunately, in practical problems linear separability often does not take place.

Various definitions of generalized separability have been proposed to satisfy the needs of applications. The first one is bilinear separability. The problem of bilinear separability consists in finding two separating hyperplanes. It is equivalent to nonconvex bilinear optimization problem [3]. The next step of the generalization is the problem of searching for p hyperplanes that separate sets $\mathcal{A}$ and $\mathcal{B}$. In this case we deal with polyhedral separability problem [4] which is equivalent to nonconvex problem with nondifferentiable functions.

Along with piecewise linear separating functions we consider the separability problems with quadratic discriminant functions $F(\cdot)$. Such problems consist in finding sphere or ellipse separating the two sets [5] and these problems are also reduced to nonconvex problems with nondifferentiable functions.

For solving the nonconvex problems we used the global search theory developed for d.c. minimization problems [6]. The algorithms elaborated on this basis are tested on specially constructed test examples and on classification problems from http://archive.ics.uci.edu/ml/datasets.html.

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GENERAL APPROACH TO MARKET NETWORK ANALYSIS

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Stock market as a complex network attract an important attention last decades. Network model of a stock market is a complete weighted graph where the nodes

are the stocks and weights of edges reflect interconnection between corresponding stocks. The most popular measure of interconnection is Pearson correlation. Network structure in this model is a characteristic of the complete weighted graph and its sub graphs. In the paper [1] the authors consider minimum spanning tree and planar maximally filtered graph as a market network structures. This approach allows to detect some hierarchical structures connecting clusters of stocks. Different approach was taken in [2] where the market graph was introduced. Market graph is obtained from the complete weighted graph by deleting all edges with weight inferior to a given threshold.

Despite the growing number of publications on the subject there is a big gap in theoretical foundations of applied techniques. In the present paper we develop market network analysis in the framework of the theory of multiple decision statistical procedures. In particular we consider the following problems:

- Choice of multivariate distribution and of measure of similarity.
- Statistical analysis of existing algorithms of network structures construction.
- Development of optimal statistical procedures.
- Statistical uncertainty of network structures.

Some results in this direction are published in the papers [3], [4].

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SOLVING CONVEX NONDIFFERENTIABLE OPTIMIZATION PROBLEMS USING THE METHOD OF SIMPLEX IMBEDDINGS

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In this paper we consider the problem

$$\begin{gathered} f_0(x) \to \min, \\ x \in X = \{x : f_i(x) \leq 0,\ i = 1, 2, ..., m\}. \end{gathered} \tag{1}$$

where f_0 – convex function, which can be nonsmooth, $f_i(x)$, $i = 1, 2, ..., m$ – convex functions, $x \in R^n$, $X \neq \emptyset$ – feasible set. We apply the method of simplex imbeddings [1] for solving the problem (1) and describe the scheme of this method in the next manner. Let simplex S_0 be given and $x^* \in S_0$ – one of the solution of the problem (1). We find the center of simplex $x^{c,0}$, then draw the cutting plane L through the point $x^{c,0}$. Note, that L has the form: $L = \left\{x : g^T (x - x^c) = 0\right\}$, where $g \in R^n$ – subgradient of the function f in the point x^c. The part of the simplex, containing the point x^*, is imbedded into new simplex with the minimal volume. Then we find the point $x^{c,1}$ and repeat the procedure until we obtain quite small volume simplex.

One of the important theoretical result, described in [1], is the estimation of simplex volumes reduction:

$$\frac{V(S^l)}{V(S^{l-1})} = \begin{cases} \frac{1}{2}, & k_l = 1, \\ \left(\frac{k_l}{k_l+1}\right)_l^k \left(\frac{k_l}{k_l-1}\right)^{l_l-1}, & 2 \leq k_l \leq n, \end{cases}$$

where $V(S^l)$ – the volume of the simplex at the iteration l, and k_l – number of saved vertexes of simplex. It is obvious that the value of volume reduction depend on the number of cut vertexes. We suggest the modification of the method based on this estimation. This modification construct cutting plane, which cut as much vertexes of simplex as possible that accelerate the convergence of the method.

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TWO FAST ALGORITHMS OF THE PROJECTION OF A POINT ONTO THE CANONICAL SIMPLEX[1]

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In this report we consider two fast algorithms of projecting a point $c \in \mathbb{R}^n$ on a standard simplex $\Lambda \subset \mathbb{R}^n$ which is defined by the following conditions:

$$\sum_{i=1}^{n} x_i = 1; \quad x_i \geq 0,\ i \in 1:n.$$

The problem in question is stated as follows:

$$\tfrac{1}{2}\sum_{i=1}^{n}(x_i - c_i)^2 \to \min_{x\in\Lambda}, \tag{1}$$

where $c_1, \ldots, c_n$ are the coordinates of the projected point c. A solution of this problem exists, and it is unique. We denote it by x^*.

A fast algorithm of finding x^* was described in [1]. The idea of the algorithm is based on algebraic analysis of the optimality conditions in Kuhn–Tucker form for the problem (1).

Earlier, the paper [2] also offering a finite algorithm of solving the problem (1) appeared. This algorithm has a geometric nature, which is confirmed in a recent work [3].

In this report we propose an improved version of description and argumentation of the algorithm from [2] and provide the results of numerical experiments comparing two fast algorithms of solving the problem (1). We notice an interesting peculiarity (cf. [4]): when one of the two algorithms takes the maximal working time, the working time of the other one is minimal.

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EXTRAGRADIENT METHODS WITH NUMBER OF ADDITIONAL STEPS [1]

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Variational inequalities are very important class of optimization problems. There are number of methods which solve them and among them gradient methods. Extragradient methods are built on basis of gradient methods, but converge with much weaker conditions on operator. Earlier there were built extragradient methods with two additional steps (see [1-2]), here generalized approach for n additional steps.

Let's specify extragradient method with n additional steps, which solves variational inequality $\langle H(z^*), z - z^* \rangle \geq 0,\ z^*, z \in \Omega$, as follows

$$\begin{array}{l} z_1^k = P_\Omega(z^k - \alpha H(z^k)), \quad z_2^k = P_\Omega(z_1^k - \alpha H(z_1^k)), \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots, \\ z_n^k = P_\Omega(z_{n-1}^k - \alpha H(z_{n-1}^k)), \quad z^{k+1} = P_\Omega(z^k - \alpha H(z_n^k)), \end{array} \tag{1}$$

where $H : \mathbb{R}^N \to \mathbb{R}^N$, Ω – closed convex set, $\Omega \subset \mathbb{R}^N$, $z^* \in \Omega^*$ – set of solutions for variational inequality, $\Omega^* \subset \Omega$, $\alpha > 0$ – parameter, P_Ω – projection operator on set Ω, z^k – known approximation of solution (or beginning point). As it seen from recurrent relations, main difference from standard extragradient method is in doing n additional steps and only with direction in last point z_n^k takes step from starting point z^k. The aim of this method is to find some point z^* from the set Ω^* of all variational inequality solutions.

Convergence of proposed iterative process (1) proved in next theorem.

Theorem. *Let's assume that variational inequality meets following conditions:*
а) Ω *– closed convex set;*
б) $H(z)$ *– monotone operator:* $\langle H(z) - H(v), z - v \rangle \geq 0,\ \forall\, z, v \in \Omega$, *which meets Lipchitz condition:* $\|H(z) - H(v)\| \leq L\|z - v\|,\ \forall\, z, v \in \Omega$;
в) set of solutions Ω^* *for variational inequality on* Ω *is not empty;*
г) $0 < \alpha < \frac{1}{\sqrt{2^{n-1}+1}L}$.
Than sequence $\{z^k\}$, *defined by recurrent relations (1), converge to some* $z^* \in \Omega^*$ *– solution of variational inequality.*

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LOCAL SEARCH PROCEDURES FOR POLYMATRIX GAMES OF THREE PLAYERS[1]

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The problem of numerical finding of a Nash equilibrium in a 3-player polymatrix game is considered, where the payoff function of each player is the sum of two bilinear terms [1]. Such a game can be completely described by six matrices, therefore, we will call it a hexamatrix game. As well known, various economical conflicts on the oligopolistic market with three players can be modeled by means of a hexamatrix game.

In order to find a Nash equilibrium point in a hexamatrix game $\Gamma = \Gamma(A_1, A_2, B_1, B_2, C_1, C_2)$ an optimization approach is used. The approach is based on the equivalence theorem for the game and a special mathematical optimization problem with a bilinear structure in the goal function ($\sigma := (x, y, z, \alpha, \beta, \gamma)$) [1]:

$$\left.\begin{array}{c} \Phi(\sigma) \stackrel{\triangle}{=} \langle x, A_1 y + A_2 z\rangle + \langle y, B_1 x + B_2 z\rangle + \langle z, C_1 x + C_2 y\rangle - \alpha - \beta - \gamma \uparrow \max\limits_{\sigma}, \\ \sigma \in D \stackrel{\triangle}{=} \{(x, y, z, \alpha, \beta, \gamma) \in I\!R^{m+n+l+3} \mid x \in S_m, \ \ y \in S_n, \ \ z \in S_l, \\ A_1 y + A_2 z \le \alpha e_m, \quad B_1 x + B_2 z \le \beta e_n, \quad C_1 x + C_2 y \le \gamma e_l\}, \end{array}\right\} \qquad (\mathcal{P})$$

where $S_p = \{u = (u_1, \ldots, u_p)^T \in I\!R^p \mid u_i \ge 0, \ \sum\limits_{i=1}^{p} u_i = 1\}$, $e_p = (1, 1, ..., 1) \in I\!R^p$, $p = m, n, l$. Components (x^*, y^*, z^*) of a global solution to problem $(\mathcal{P})$ are Nash equilibrium point, and $(\alpha_*, \beta_*, \gamma_*)$ are payoffs of players 1, 2, and 3, respectively, of the game Γ [1].

We propose to solve problem $(\mathcal{P})$ by means of the Global Search Theory in nonconvex problems with (d.c.) functions of A.D. Alexandrov [2]. According to the Theory one of the basic element of a global search is a local search, which takes into account the structure of the problem under scrutiny.

To implement a local search in problem $(\mathcal{P})$, let us apply the idea, first, of splitting variables in several groups, and, after that, of consecutive solving of partial LP problems with respect to groups of variables [2]. This idea has previously demonstrated its efficiency in problems with a bilinear structure. By means of various splitting ways of variables, 12 variants of local search procedures for problem $(\mathcal{P})$ were developed. The convergence of these procedures is investigated, practical stopping criteria are proposed, and computational testing of the local search in random generated hexamatrix games is implemented.

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DUAL BARRIERS AND REGULARIZATION IN NUMERICAL ANALYSIS OF IMPROPER LINEAR PROGRAMS OF THE FIRST KIND[1]

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We consider the primal linear program

$$\min\{(c,x)\colon Ax = b,\ x \geq 0\} \tag{1}$$

and the dual one

$$\max\{(b,y)\colon A^T y \leq c\} \tag{2}$$

under the only assumption that the feasible set Y of (2) is nonempty. It means that the program (1) may be either solvable (if its constraints are consistent) or improper of the 1-st kind [1] (if it is not). In the second case the program (1) can be transformed to a solvable one by means of any suitable correction of the right-hand-sides of its constraints.

Following [1] one can put program (1) into parametric family of linear programs like

$$\min\{(c,x)\colon Ax = b - u,\ x \geq 0\} \tag{3}$$

and denote the set of all u which guarantee the consistence of constraints of (3) (and solvability of (3)) by Ω. We determine the optimal correction vector as

$$u_0 := \arg\ \min\{\|u\|\colon u \in \Omega\};$$

where $\|\cdot\|$ is Euclidean norm. It is clear that in a proper case $u_0 = 0$.

Let X_0 be the optimal set of (3) corresponding to $u = u_0$; $e = (1,\ldots,1)$, $A_1,\ldots,A_n$ be the columns of the matrix A. To combine the process of correction of (1) with the process of optimization of its criterium we suggest to use the standard logarithmic barrier function for the dual problem (2) supplemented with quadratic regularization term:

$$B(\varepsilon; y) = (b,y) + \varepsilon_1 \sum_{i=1}^{n} \ln(c_i - (A_i, y)) - \frac{\varepsilon_2}{2}\|y\|^2. \tag{4}$$

Theorem. *If the set Y has at least one inner point then for any pair $\varepsilon = (\varepsilon_1,\ \varepsilon_2) > 0$ there exists the unique vector $\hat{y}_\varepsilon$ that belongs to the interior of Y and maximize the function* (4). *More over,*

$$\hat{x}_\varepsilon := \varepsilon_1 \mathrm{diag}\Big(c - A^T \hat{y}_\varepsilon\Big)^{-1} e \to X_0, \qquad \varepsilon_2 \hat{y}_\varepsilon \to u_0$$

as $\varepsilon = (\varepsilon_1,\ \varepsilon_2) \to +0$.

Note that second order optimization methods may be used for maximization of a smooth concave function like $B(\varepsilon;\cdot)$.

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ON THE RESIDUAL METHOD FOR THE CORRECTION OF CONTRADICTORY PROBLEMS OF CONVEX PROGRAMMING

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Consider the convex programming (CP) problem

$$\min\{f_0(x) \mid x \in X\}, \tag{1}$$

where $X = \{x \mid f(x) \leq 0\}$, $f(x) = [f_1(x), \ldots, f_m(x)]$, $f_i(x)$ are convex functions defined on $\mathbb{R}^n$ $(i = 0, 1, \ldots, m)$. The CP problems with contradictory constraints $(X = \varnothing)$ represent [1] an important class of improper CP problems (ICPP).

The improper problems often appear because of the errors in the initial data, that is connected with the stability of the optimal elements. Similar problems are an object for the theory of ill-posed optimization problems. This fact makes it possible to consider the standard regularization methods of the ill-posed models for the analysis of improper problems. In this report the possibility of the use of the residual method [2] for the approximation of the ICCP is investigated.

In the residual method the following similar problem is considered along with (1):

$$\min\{\|x\|^2 \mid x \in X \cap M_\delta\}, \tag{2}$$

where $M_\delta = \{x \mid f_0(x) \leq \delta\}$, $\delta \in \mathbb{R}^1$, $\delta \geq f^*$, f^* is the optimal value of problem (1). The problem (2) has a unique optimal point x^*_δ, and it is easy to verify, that sequence x^*_δ converge to the normal solution of the problem (1) as $\delta \to f^*$.

If the set X is empty, then the problem (2) is also the ICCP. In this case we register the restrictions of the problem (2) by a penalty function. There are considered the method of the quadratic penalty function and the exact penalty function method. Then instead (2) there are investigated the problem

$$\min_x\{F_\delta(x, r) = \|x\|^2 + \rho\|f^+(x)\|^2 + \rho_0(f_0(x) - \delta)^{+^2}\}, \tag{3}$$

and the problem

$$\min_x\Big\{\Phi_\delta(x, r) = \|x\|^2 + \rho\sum_{i=1}^{m} f_i^+(x) + \rho(f_0(x) - \delta)^+\Big\}, \tag{4}$$

respectively, where $r = [\rho, \rho_0] \in \mathbb{R}^2$, $r > 0$, $\delta \in \mathbb{R}^1$. The both problems (3), (4) have a unique solution for any r, δ and also for $X = \varnothing$. This reason admits to use the functions $F_\delta(x, r)$ and $\Phi_\delta(x, r)$ for the analysis of the ICCP.

In the report specific attention is paid to finding estimates, that characterize the convergence of the solutions (3) and (4) to a approximate solution of the ICCP.

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1d AND 2d ELLIPSOIDS IN CONVEX PROGRAMMING

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In this report, we discuss topic of „building of an algorithm, which would be practically no less efficient than r-algorithm and would be as substantiated as ellipsoid method“, initiated by N.Z. Shor in 1982. Ellipsoid method uses space dilation operator $R_\alpha(\xi) = I + (\alpha - 1)\xi\xi^T$. With this, a minimal volume ellipsoid, containing half-sphere in n-dimensional Euclidean space, is trasformed into a new sphere in one space dilation(1d-ellipsoid).

Lemma [1]. *Let B_k be nondegenerate $n\times n$-matrix, with $\|B_k^{-1}(x_k - x^*)\| \le r$; g_1 and g_2 – n-dimensional vectors, complying with $(x_k - x^*, g_1) \ge 0$ and $(x_k - x^*, g_2) \ge 0$. If the condition $-\|B_k^T g_1\|\|B_k^T g_2\| < (B_k^T g_1, B_k^T g_2) < 0$ is satisfied and matrix B is recalculated according to the rule*

$$B_{k+1} = B_k R_{\beta_1}\left(\frac{\xi - \eta}{\|\xi - \eta\|}\right) R_{\beta_2}\left(\frac{\xi + \eta}{\|\xi + \eta\|}\right), \quad \xi = \frac{B_k^T g_1}{\|B_k^T g_1\|},\ \eta = \frac{B_k^T g_2}{\|B_k^T g_2\|},$$

where $\beta_1 = \sqrt{1 + (\xi, \eta)}$ u $\beta_2 = \sqrt{1 - (\xi, \eta)}$, then matrix B_{k+1} is nondegenerate and has the following properties:

(i) $\|B_{k+1}^{-1}(x_k - x^*)\| \le r$; (ii) $\det(B_{k+1}) = \det B_k \sqrt{1 - (\xi, \eta)^2}$; (iii) $(B_{k+1}^T g_1, B_{k+1}^T g_2) = 0$.

Lemma has the following interpretation: Property (i) assures point x^* location withing the next 2d-ellipsoid, and property (ii) means its volume decrease in comparison with that of previous ellipsoid. Property (iii) assures use of anti-ravine method like one used in r-algorithms. This means that subgradients with obtuse angle in current variable space become orthogonal in transformed space, that allows to improve level surfaces of ravine function. Coefficients of space dilation in direction of difference of normalized subgradients and in direction of sum of normalized subgradients are defined by angle between subgradients. The more obtuse the angle, the more space dilation coefficient in the direction of difference of two normalized subgradients.

2d-ellipsoid can be used for construction of accelerated ellipsoid methods for wide class of problems: convex programming problems, finding saddle points of concave-convex functions, special cases of solution of variational inequalties, special classes of problems of linear and non-linear complimentarity. For these methods, one can assure speed of convergence close to one of r-algorithms. This is confirmed by subgradient methods with space transformation for finding the minimum point of a convex function with a priori knowledge of function value at the minimum point [2]. They proved to be effective while working with ravine functions.

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MAXIMUM CONSISTENCY METHOD FOR DATA FITTING UNDER INTERVAL UNCERTAINTY

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For the linear regression model $b = a_1x_1 + a_2x_2 + \ldots + a_nx_n$, we consider the problem of data fitting under interval uncertainty. Let an interval $m \times n$-matrix $\boldsymbol{A} = (\boldsymbol{a}_{ij})$ and an interval m-vector $\boldsymbol{b} = (\boldsymbol{b}_i)$ represent, respectively, the input data and output responses of the model, such that $a_1 \in \boldsymbol{a}_{i1}$, $a_2 \in \boldsymbol{a}_{i2}$, $\ldots$, $a_n \in \boldsymbol{a}_{in}$, $b \in \boldsymbol{b}_i$ in the i-th experiment, $i = 1, 2, \ldots, m$. It is necessary to find the coefficients that best fit the above linear relation for the data given.

A family of values of the parameters x_i is called *consistent* with the interval data $(\boldsymbol{a}_{i1}, \boldsymbol{a}_{i2}, \ldots, \boldsymbol{a}_{in})$, $\boldsymbol{b}_i$, $i = 1, 2, \ldots, m$, if, for every index i, there exist such point representatives $a_{i1} \in \boldsymbol{a}_{i1}$, $a_{i2} \in \boldsymbol{a}_{i2}$, $\ldots$, $a_{in} \in \boldsymbol{a}_{in}$, $b_i \in \boldsymbol{b}_i$ that $a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n = b_i$. The set of all the parameter values consistent with the data given form a *parameter uncertainty set*. As an estimate of the parameters, it makes sense to take a point from the parameter uncertainty set providing that it is nonempty. On the contrary, if the parameter uncertainty set is empty, then the estimate should be a point where maximal "consistency" (in a prescribed sense) with the data is achieved.

The parameter uncertainty set, as defined above, is nothing but the solution set $\Xi(\boldsymbol{A}, \boldsymbol{b})$ to the interval system of linear equations $\boldsymbol{A}x = \boldsymbol{b}$ introduced in interval analysis: $\Xi(\boldsymbol{A}, \boldsymbol{b}) = \{\, x \in \mathbb{R}^n \mid Ax = b \ \text{ for some } A \text{ from } \boldsymbol{A} \text{ and } b \text{ from } \boldsymbol{b} \,\}$. For the data fitting problem under interval uncertainty, we propose, as the consistency measure, the values of the *recognizing functional* of the solution set $\Xi(\boldsymbol{A}, \boldsymbol{b})$ which is defined as

$$\operatorname{Uss}(x, \boldsymbol{A}, \boldsymbol{b}) = \min_{1 \le i \le m} \left\{ \operatorname{rad} \boldsymbol{b}_i + \sum_{j=1}^{n} (\operatorname{rad} \boldsymbol{a}_{ij})\, |x_j| - \left| \operatorname{mid} \boldsymbol{b}_i - \sum_{j=1}^{n} (\operatorname{mid} \boldsymbol{a}_{ij})\, x_j \right| \right\},$$

where "mid" and "rad" mean the midpoint and radius of an interval. The functional Uss "recognizes" the points of $\Xi(\boldsymbol{A}, \boldsymbol{b})$ by the sign of its values: $x \in \Xi(\boldsymbol{A}, \boldsymbol{b})$ if and only if $\operatorname{Uss}(x, \boldsymbol{A}, \boldsymbol{b}) \ge 0$. Additionally, Uss has reasonably good properties as a function of x and $\boldsymbol{A}$, $\boldsymbol{b}$.

As an estimate of the parameters in the data fitting problem, we take the value of $x = (x_1, x_2, \ldots, x_n)$ that provides maximum of the recognizing functional Uss (Maximum Consistency Method). Then, if the parameter uncertainty set is nonempty, we get a point from it. Otherwise, if the parameter uncertainty set is empty, we get a point that still has maximum possible consistency (determined by values of the functional Uss) with the data given.

In our work, we discuss properties of the recognizing functional Uss, interpretation and features of the estimates obtained by the Maximum Consistency Method. Also treated is correlation with the other approaches to data fitting under interval uncertainty.

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A PARAMETRIC VERSION OF GENERALIZED LINEAR EXCHANGE MODEL[1]

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The participants' set of the generalized linear exchange model include alongside consumers also firms supplying the market with the additional volumes of commodities. Each firm is characterized by some total financial liability of the production which is to be kept providing a minimal discontent with the production plan. The consumers have some initial money stock and initial stock of products.The purchase and sale of products for all participants (consumers and firms) are carried out by common prices. Choosing a purchase vector of the products, the consumer tends to maximize his linear utility function. The concept of an equilibrium for this model is introduced by analogy with the linear exchange model.

In the parametric version of the model we study the equilibrium change when the consumers money stock is a linear function of one parameter. An finite algorithm based on polyhedral complementarity approach is proposed.

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DUAL PARALLEL ALGORITHMS FOR PROJECTION OF A POINT ONTO THE SET

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We consider a projection problem as a quadratic optimization problem with linear inequality constraints [1]. After dual equivalent transformation the parallel version of modified nonliear Jacobi algotithm is applyed [2]. The use of the alrorithm is to solve simple one-dimensional quadratic problems.

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A PROJECTIVE SEPARATING PLANE METHOD WITH ADDITIONAL CLIPPING AND ITS APPLICATION TO THE TRANSPORTATION PROBLEMS

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We consider a separating plane algorithm with additional clipping [1]. The algorithm is used for solving the unconstrained non-smooth convex optimization problem. The latter problem can be reformulated as the computation of a conjugate function value at the origin [2].

The algorithm was successfully applied to the solution of transportation problems with two-sided constraints. The presence of such constraints is often a problem for the method of potentials and the simplex method. The linear programming task was replaced by a linear variety projection task. Results of numerical experiments are presented.

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A cutting method with updating approximating sets and its combination with other algorithms

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We propose a conditional minimization method which belongs to the class of cutting methods with approximation of the epigraph of the objective function (e. g., [1]). One of its principal features consists in periodically dropping of cutting planes, where there are some advantages from the practical viewpoint.

We solve a minimization problem of a convex function $f(x)$ on a closed convex set $D \subset R_n$. The proposed method for solving the problem consists in the following. Choose a point $v^j \in intE$ for each $j \in J = \{1, \dots, m\}$, where $E = \{(x, \gamma) \in R_{n+1} : x \in R_n,\ \gamma \geq f(x)\}$, construct a closed convex set $M_0 \subset R_{n+1}$ which contains E, put numbers $\varepsilon_k > 0$, $k \in K = \{0, 1, \dots\}$, $\varepsilon_k \to 0$, $k \to \infty$, $\overline{\gamma} \leq f^* = \min\{f(x) : x \in D\}$, set $i = 0$, $k = 0$.

1. Find a decision (y_i, γ_i) of the problem

$$\min\{\gamma : (x, \gamma) \in M_i,\ x \in D,\ \gamma \geq \bar{\gamma}\},$$

where $y_i \in R_n$, $\gamma_i \in R_1$

2. If $f(y_i) - \gamma_i > \varepsilon_k$, then put $Q_i = M_i$, $u_i = y_i$. Otherwise choose a closed convex set $Q_i \subset R_{n+1}$ which contains E and a point $x_k \in D$ such that $f(x_k) \leq f(y_i)$, put $\sigma_k = \gamma_i$, increase the value of k by one.

3. For each $j \in J$ choose a point $z_i^j \notin intE$ in an interval $(v^j, (u_i, \gamma_i))$ according to some rule and construct a finite set A_i^j of normalized general support vectors for the set E in the point z_i^j.

4. Put $M_{i+1} = Q_i \bigcap\limits_{j \in J} \{w \in R_{n+1} : \langle a, w - z_i^j \rangle \leq 0\ \forall a \in A_i^j\}$, increase the value of i by one, and go to Step 1.

We prove an optimality theorem for the point y_i. We establish that the method constructs the basic sequence $\{x_k\}$, $k \in K$, with the sequence of auxiliary points y_i, $i \in K$, and we obtain the equality $\lim\limits_{k \in K} f(x_k) = f^*$ for the sequence $\{x_k\}$, $k \in K$.

In case of the strongly convex function $f(x)$ with the parameter μ we obtain $||x_k - x^*|| \leq \sqrt{\varepsilon_k / \mu}$, where x^* is a solution of the problem.

We discuss construction ways of points ε_k and sets M_0, Q_i, A_i^j. We show how due to choosing sets Q_i which differ from M_i, in particular $Q_i = R_{n+1}$, the method allows to update approximating sets M_{i+1} by dropping accumulated any number of cutting planes. Note that in case of $x_k \neq y_{i_k}$ choosing condition of the point x_k allows to combine this method with other algorithms saving its convergence, and to use parallel computings for finding x_k.

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OPTIMAL DECISIONS SEARCHING IN MULTICRITERIA GROUP CHOICE PROBLEM USING MAJORITY RELATION[1]

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A multicriteria group choice problem is considered in the paper, it includes: 1) a set of feasible decisions $X \subseteq \mathbb{R}^n$; 2) a vector criterion $f = (f_1, \ldots, f_m)$ defined on the set X; 3) asymmetric binary relations $\succ_i$, $i = \overline{1, N}$, of decision makers (DM) defined on the set $Y = f(X)$. Vector criterion f reflects general goals of DMs group, binary relation of each DM reflects individual preferences, which are given by «quanta» of information [1]. It indicates compromise between two components of criteria f. Conditions about binary relation $\succ_i$ for all $i = \overline{1, N}$ and vector criterion f are applied [1]. The majority relation $\succ$ is considered: for arbitrary vectors $y^{(1)}, y^{(2)} \in \mathbb{R}^m$ the relation $y^{(1)} \succ y^{(2)}$ is valid, if the relations $y^{(1)} \succ_i y^{(2)}$ are true for at least half numbers i from $\{1, \ldots, N\}$. It is proved that the relation $\succ$ is a cone relation with a cone $K = \bigcup_{i=1}^{C_N^p} \bigcap_{l=1}^{p} K_{il}$, where $p = N/2$, if p is even, $p = [N/2] + 1$, if p is odd, $[a]$ — integer part of number a, $\{K_{i1}, \ldots, K_{ip}\} \subset \{K_1, \ldots, K_N\}$. It is shown that $0_m \notin K$, $\mathbb{R}^m_+ \subseteq K$, and in spite of cones $K_1, \ldots, K_N$ are convex cone K, in general, is not the same.

Let optimal decisions of multicriteria group choice problem be the set of nondominated vectors $\mathrm{Ndom}_K(Y) = \{y^* \in Y \mid \nexists y \in Y : \ y - y^* \in K\}$ according to cone K. In case $K = \mathbb{R}^m_+$, the equality $\mathrm{Ndom}_K(Y) = P(Y)$ holds, where $P(Y)$ is the Pareto set. When cone K is not convex the majority relation $\succ$ is not transitive. Thus, it is necessary to specify the convex part $\hat{K}$ of cone K, which defines the transitive part $\succ_{tr}$ of the majority relation $\succ$. Then, in fact, the set of nondominated vectors $\mathrm{Ndom}_{\hat{K}}(Y)$ should be searched instead of set $\mathrm{Ndom}_K(Y)$.

The case of group of three DMs and two components of criteria is considered. Each DM gives one «quantum» of information and two «quanta» of information. It is shown how to specify the convex part $\hat{K}$ of cone K, and construct the set of nondominated vectors $\mathrm{Ndom}_{\hat{K}}(Y)$: it is the Pareto set $\hat{P}(Y) = f(P_g(X))$ in multicriteria problem with «new» vector criterion g and the set of feasible decisions X, where the components of criteria g are linear combinations of the components of criteria f. And $\mathrm{Ndom}_{\hat{K}}(Y) \subseteq P(Y)$, i. e. the reduction of initial optima set $P(Y)$ is derived. However, it is shown the case where cone $\hat{K}$ is chosen ambiguously that leads to ambiguity of the set of nondominated vectors $\mathrm{Ndom}_{\hat{K}}(Y)$.

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CONVERGENCE OF THE EXTRAGRADIENT METHOD IN A FINITE NUMBER OF ITERATIONS [1]

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The article deals with double-step extragradient method for solving variational inequalities. It is reported that convergence in a finite number of steps uder the severity conditions.

To solve the variational inequality it is means that to find a vector $z^* \in \Omega$ which satisfies the following conditions:

$$\langle H(z^*), z - z^* \rangle \geq 0, \quad \forall\, z \in \Omega, \tag{1}$$

Where $H : \mathbb{R}^n \to \mathbb{R}^n$, Ω – convex closed set, $\Omega \subset \mathbb{R}^n$, $z^* \in \Omega^*$ – set of solutions for the variational inequality, $\Omega^* \subset \Omega$.

The convergence of the two-step extragradient method for solving $z^* \in \Omega^*$ variational inequalities (1) with monotone operator under the Lipschitz condition with a constant $L > 0$, is provided by the value of the step α which satisfies the following conditions $0 < \alpha < \frac{1}{\sqrt{3}L}$ [1].

Let's consider the additional condition for continuous monotone operator $H(z)$ which is a condition of sharpness. It is means [2] : The condition

$$\langle H(z), z - z^*(z) \rangle \geq \gamma \|z - z^*(z)\|, \quad \forall\, z \in \Omega, \quad z^*(z) = P_{\Omega^*}(z). \tag{2}$$

is performed for the variational inequality, where $\gamma > 0$.
More strict condition $\langle H(z), z - z^* \rangle \geq \gamma \|z - z^*\|$ of sharpness, which assumes the exist the unique solution $z^* \in \Omega$, that was introduced in [3]. The sharpness condition (2) for variational inequalities (1) with potential mapping $H(z) = \nabla f(z)$ is a known condition of sharp minimum $f(z) - f(z^*(z)) \geq \gamma \|z - z^*(z)\|$, $\forall\, z \in \Omega$, for the corresponding aim for convex optimization with convex closed set of solutions $\Omega^* \subset \Omega$ [4].

The convergence of the sequence $\{z^k\}$ for a variational inequality (1) under the condition of sharpness is showed. The sequence $\{z^k\}$ is defined by the recurrence relations of the two-step extragradient method and is converged to the solution $z^* \in \Omega^*$ of the variational inequality (1) for the finite number of iterations.

A similar result is obtained in the one-step method extragradient [3] under the condition of sharpness.

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OPTIMAL CONTROL

NON-LOCAL METHODS OF SOLUTIONS OF LINEAR-QUADRATIC OPTIMAL CONTROL PROBLEM

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The talk views the convex linear-quadratic problem

$$\Phi(u) = \langle c, x(t_1)\rangle + \frac{1}{2}\langle x(t_1), Dx(t_1)\rangle + \frac{1}{2}\int_T (\langle x(t), Q(t)x(t)\rangle + u^2(t))dt,$$

$$\dot{x} = A(t)x + b(t)u, \quad x(t_0) = x^0,$$

$$V = \{u(\cdot) \in PC(T) : \ u(t) \in [u^-, \, u^+], \ \ t \in T\}.$$

For this task we present two independent methods of improvement of admissible controls, similar in complexity of realization (the price of one improvement - two of the Cauchy problem). It turns out that both methods can be naturally joined and get a combined method, which is within the same complexity provides dual improvement in functionality (the price of each improvement is one of the Cauchy problem). This method seems to be the most effective procedure of the numerical solution of the task.

Let k-iteration have the valid process $(u^k(t), x^k(t)), t \in T$. We will find the solution $p^k(t), \ t \in T$ with a combined system, together with the interim control

$$\dot{p} = -A(t)^T p + Q(t)x^k(t) - \Psi(t)b(t)(u_*(p,t) - u^k(t)),$$

$$p(t_1) = -c - Dx^k(t_1),$$

$$v^k(t) = u_*(p^k(t), t), \quad t \in T.$$

From the vector-function and auxiliary control

$$p(t, v^k, x) = p^k(t) + \Psi(t)(x - x^k(t)),$$

$$w^k(x,t) = u_*(p(t, v^k, x), t).$$

We will find the solution $x^{k+1}(t), \ t \in T$ of the phase system in conjunction with the control

$$u^{k+1}(t) = w^k(x^{k+1}(t), t), \quad t \in T.$$

The iteration is completed.

Thus, the iteration process of the combined method results in double improvement in the functional $\Phi(u^{k+1}) \le \Phi(v^k) \le \Phi(u^k)$, and each improvement comes at the price of one solution of the Cauchy problem.

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APPLICATION OF OPTIMAL CONTROL METHODS TO A PROBLEM OF COMPOSITE CIRCULAR PLATES DESIGN[1]

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Thin plates are widely used as the most important elements of different high-duty structures. A stringency of requirements to these structures being significantly higher than to regular structures, it brings about a necessity to apply novel composite materials (CM) when manufacturing them combining high strength and rigidity of CM with their other valuable qualities, on the one hand, and, on the other hand, developing optimal and rational design of composite structures in order to identify and use their potential capabilities to the fullest extent.

The report discusses the results of how computing optimal control methods [1] are applied to solve the problems of designing a minimum weight plates under their limited load bearing capacity. The statement of the problem is based on application of structural models of CM [2] which allowed us to describe behavior of plates with a use of a functional relationship between parameters of a material and characteristics of each CM component (such as a binder and fiber reinforcement). Structural and geometric characteristics were used as optimization parameters, while a radius of designed circular plates was an independent variable. The study of laid down optimal control problems showed that even with such a small dimensionality (2 phase variables) several factors manifest themselves making its numerical analysis more complicated: multiextremal character, rigidity of differential equations system, ill-conditioning of objective functionals and others. OPTCON-A Software [1] was used to solve optimization problems. The results of computational experiments are presented in the report.

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THE OPTIMAL AERODYNAMIC SHAPE OF BODIES WITH MINIMAL HEAT TRANSFER

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Optimal aerodynamic design of space apparatus is one of the most interesting and promising fields of the applications of the optimal control methods. It is known that spacecraft shape has a great influence on its aerodynamic and heat characteristics. So, one of the efficient methods of solving problem of minimizing the aerodynamic drag and heat fluxes to the apparatus surface is a choice of an optimal body shape.

In addition to improving traditional configurations (axis-symmetric bodies and flat wings) the new shapes of bodies with optimum thermophysical and aerodynamic characteristics are now under investigation. It is known that bodies with circular transversal sections are not optimal with viewpoint of drag minimization. In particular, in [1] it was shown that the drag of star-like bodies is less then the drag of equivalent bodies of the revolution. Note, that identical results take place in the optimal design of three-dimensional bodies with minimal surface heat. In paper [2] the problems of minimization of total radiation heat fluxes along flight trajectory in atmosphere for 3-dimensional bodies were investigated.

However, the review of scientific references on this problem [2] has shown that in spite of wide research of optimization problems of super- and hypersonic aerodynamics many questions connected with the joint influence of heat fluxes and drag on the shape of the 3-dimensional bodies with non-circular transversal sections are not well studied.

This paper presents investigation of a new multi-criterion problem of minimization of the drag and heat fluxes to the surfaces of three-dimensional bodies. In a class of slender bodies possessing homothetic property the initial optimization problem may be reduced to the problem of searching optimal transverse contours. From mathematical point of view this problem is isoperimetric variational one with two cost functionals. The "ideal point"method is applied for solution of such optimization problem.

The investigation of the problem of determining the optimal transversal contour has shown that there exists a class of variational solutions composed of n identical cycles. A distinguishing feature of the suggested approach is that the minimization procedure includes not only a search for each extremal segment but also the number of these segments. This leads to the additional optimizing condition on the number of cycles. Joint integration of the Euler-Lagrange equation and the condition mentioned above has permitted the author to obtain three classes of analytical solutions.

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TURNPIKE SOLUTION IN MEDICAL-ECOLOGICAL-ECONOMIC PROBLEMS ON THE EXAMPLE OF ULAN BATOR CITY[1]

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Applying mathematical modeling for studying economic, environmental factors and morbidity is an important step in the system analysis of the social development problems. Estimation and forecasting of medical-ecological-economic state of a region is actual and important task which helps to solve problems of life quality and sustainable development. Creating dynamic mathematical models of medical-ecological-economic systems and multivariate calculations let us to analyze the influence of various factors on the development prospects of the region. Similar studies were conducted for the medical-ecological-economic model of enterprises in the city. [1] At present the development of mathematical models to assess and predict medical-ecological-economic state of Ulan Bator is carrying out.

The next step in applying medical-ecological-economic models is formulating optimal control problems. In this study solutions of optimal control problems are found in the form of the magistrals (turnpikes) [2]. Turnpike solution determines the optimal trajectory, which does not depend directly on the boundary conditions. In order to obtain a solution satisfying the given initial and final conditions turnpike solution is approximated, for example, by a sequence of linear functions in times of entry and exit from the magistral. To find the turnpike solutions method of multiple maxima [3] is used.

In order to carry out the search procedure of turnpike solution we have to formulate criterion of optimality, the restrictions on the model variables and introduce additional assumptions for some elements of the model (eg, dynamics of some variables represent as exogenous functions of time). For such medical-ecological-economic optimal control problems we consider various criterions of optimality.

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INNOVATIVE APPROACH TO RISK MANAGEMENT INTEGRATED BUSINESS

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Currently, risk management methodology takes into account the risk management practices at the level of inter-firm interactions. Particular attention is paid to the risks of inter-firm interactions, due to changes in global economic conditions and the growth of economic integration processes.

The essence of the approach is that the risks are considered to places of origin at each stage of the circuit current assets of industrial enterprises, and the integral value of the risks caused by the parameters and conditions of interaction with suppliers of material resources and buyers of finished products.

Thus, one of the most important tasks of an innovative approach to risk management is integrated enterprises to find the optimal ratio of interaction with main contractors to ensure the optimal level of risk of an industrial enterprise.

To solve this problem it was necessary to abandon the use of the traditional approach to classification and definition of risk for subsequent detailed analysis. It is possible to develop innovative internal structure of industrial enterprise risks proposed for a more detailed analysis of accounting and identify the risks in terms of integration processes.

Proposed an innovative approach to optimize the level of risk is divided into six stages.

At the first stage the list of possible risks, divided into nine major groups: the risks associated with supplier; risks processes "Purchase of material resources" , "Storage, storage, internal transportation resources" , "Production" , "Storage. Storage. Domestic transportation" , "sales of finished products" , "transportation of finished products" , "Financing of current assets" ; risks associated with the buyer.

The second step is to determine the magnitude of potential losses and the probability of their occurrence. To do this, provided the necessary statistical and operational information for determining the values of losses and the probability of their occurrence. Then, the grouping is performed and the calculation of losses that can occur when operation of the focus for each of the types of risks. In the third stage the mathematical expectation of losses in stages circuit current assets. We calculate milestones expectation of losses for each type of risk.

In the fourth step, the analysis of the mathematical expectation of losses.

In the fifth stage the integral indicator of the level of risk collectively integrating enterprises, in particular integral indicator of the level of risk of the supply chain such as "Supplier Resources – Manufacturer – Buyer the finished product" .

The value obtained allows to relate the cumulative (integrated) risk indicators with the focus of the company's turnover and profit and risk attributed to one of the following categories: minimal, valid, critical, catastrophic.

At the final stage, the search for the optimal parameters of the supply chain to reduce the integral value of risk.

OPTIMAL CONTROL OF A MARKETING CAMPAIGN ACTIVITIES [1]

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Optimization marketing campaign problem is considered in this report. In this work "Activity"means any event which has fixed date and paid participation. For example: concert, conference, flight on a plane.

Organiser's goal is to get the biggest profit from holding the conference. Number of potential buyers is not known priori. Moreover, demand for tickets depends on their cost and on potential buyers awareness. Thus, organiser can manages marketing campaign on the whole interval of time from start to event day in two ways:

1. To change the ticket cost.
2. To invest in promotion campaign.

Promotion campaign is considered like collection of different promotion tools. Number of people is attracted by promotion tool is considered like random variable. However, the dependence of the average number of attracted clients from a number of investments in advertising tool is known.

Also the model considers that the time at which a sale is made affects the probability of purchase. The closer the event, the greater the probability of purchase (ceteris paribus) the potential customer.

In the model, we consider the discrete time selling tickets, so for each time organizer sets the ticket price and allocates funds to various promotional tools. Time of sale of the ticket and the ticket price are the main factors affecting the probability of buying a ticket.

Because objective function (profit) depends on the random variables, it can not be optimized explicitly. To solve the problem, a quantile criterion is used. Quantile characterizes the maximum profit that is obtained organizer, with a given probability [1].

The considered problem solved with the following natural constraints:

1. Maximum number of tickets sold shall not exceed the number of seats at the event. Restriction is probabilistic, because the number of buyers is random.
2. Money spent on an advertising campaign and general expenses must not exceed the campaign budget. Budget restriction is deterministic.

In the report an algorithm for solving the problem in the class of program strategies is offered, i.e. prices and costs are determined for the entire planning period.

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ALGORITHMS FOR QUASIUNIFORM APPROXIMATION OF THE REACHABLE SET OF THE CONTROLLED SYSTEM WITH TWO AND THREE PHASE VARIABLE [1]

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The paper considers the algorithms for the search of accessible points that fills volume of the reachable set in a quasi-uniform way, and thus, unlike the method of stochastic approximation resulting in a cloud of points uniformly (with some accuracy) approximating the set even with a small number of points. The proposed algorithms are similar to the method of "deep holes"[1] and requires multiple solution of auxiliary optimization problems for adding points to the approximating set.

Let's consider nonlinear controlled systems $\dot{x} = f(x(t), u(t), t)$ with the initial conditions $x(t_0) = x_0$ and control constraints $\underline{u} \leq u(t) \leq \overline{u}$ on $t \in [t_0, t_1]$. Algorithms for constructing approximations include sequential addition of points $x^i(t_1)$ to the set $\{x^n\}$ obtained at the previous stages. In the first variant of the algorithm the problem of searching of each additional point consits in solving the maximin optimal control problem $x^* = \max_u \min_{i=\overline{1,n}} \rho_i$, where $\rho_i = ||x^i - x(t_1)||_2$. In order to use optimization methods designed for smooth functionals for solving this problem it was proposed several versions of smooth approximation of discrete maximin functional. An intermediate solution will be obtained after completing each stage of optimization, so elapsed computing time can be viewed as an additional criterion for stop this algorithm. The second algorithm is based on the minimization of a continuous function $x^* = \min_u \sum_{i=1}^{nb} S(\rho_i)$ that depends of the distance between the points and determines to be equal to zero if ρ_i more than the desired threshold value, equal to a sufficiently large number in $\rho_i = 0$, and monotonically decrease in the gap. The fact that the lower bound of optimal value of the functional is known in advance greatly reduces computational time while using stochastic algorithms of global optimization algorithm. The algorithms stops when it is impossible to add a point that satisfies the uniformity condition. If the number of approximation points are the order of 10^2 results of the algorithms are close, but with the successful choice of the function the second algorithm will provide a solution quickly, while, with a smaller number of approximating points, the first algorithm will produce much more reliable results. The boundary points of its resulting set will be the boundary points of the reachable set with an accuracy of solving optimization problems, in particular that is because it's no necessary to setup algorithm parametres. Computational experiments with the proposed algorithms are performed for two-dimensional and three-dimensional problems, the results obtained by visualization of test reachable sets.

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EXACT PENALTIES IN A PROBLEM OF OBTAINING THE OPTIMAL SOLUTION OF A DIFFERENTIAL INCLUSION[1]

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In this report a differential inclusion

$$\dot{x} \in F(x,t), \tag{1}$$

is considered, where $F(x,t)$ — given multivalued mapping in R^n, $x(t)$ — n-dimensional vector function of phase variables, $t \in [0,T]$. It is assumed that each point in time $t \in [0,T]$, and each phase point $x \in R^n$ the function $F(x,t)$ associates a convex compact. From each initial point

$$x_0 = x(0) \tag{2}$$

of the inclusion (1) comes out a whole family of trajectories, and the problem is to find such solutions, that deliver minimum to the integral functional

$$I = \int_0^T f_0(x,t)dt. \tag{3}$$

In [1, 2] some classical results are given which extend known Pontryagin's maximum principle for differential inclusions. The maximum principle is obtained under sufficiently stringent assumptions, in particular, provided that the support function $c(F(t,x),\psi)$ of the multivalued mapping $F(t,x)$ is differentiable on vector of phase coordinates.

The present report attempts to generalize these results to the case when the differentiability on x of the support function $c(F(t,x),\psi)$ is not assumed. This generalization is carried out using the apparatus of support functions [3] and exact penalty functions [4, 5]. Nonsmooth optimization methods [6] give opportunity to obtain the necessary minimum conditions of the functional (3) under constrains (1), (2).

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OPTIMAL CONTROL OF TRAJECTORY BUNDLES

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We consider an optimal control problem of the form

$$\frac{dx}{dt} = f(t,x,u), \quad t \in T_0 = [0,T], \tag{1}$$

$$\frac{\partial \rho}{\partial t} + \left\langle \frac{\partial \rho}{\partial x}, f(t,x,u) \right\rangle = -\rho \operatorname{div}_x f(t,x,u), \quad t \in (0,T),\ x \in \Omega, \tag{2}$$

$$x(0) \in M_0, \tag{3}$$

$$\rho(0,x) = \rho_0(x), \quad x \in \overline{\Omega}, \tag{4}$$

$$u(t) \in U,\ t \in T_0, \tag{5}$$

$$I(u) = \int_{T_0} \int_{M_{t,u}} F(t,x,\rho(t,x))\, dxdt + \int_{M_{T,u}} \varphi(x,\rho(T,x))\, dx \to \inf . \tag{6}$$

Here, $x : T_0 \to \mathbf{R}^n$, $\rho(t,x) : T_0 \times \overline{\Omega} \to \mathbf{R}^1$, $u : T_0 \to \mathbf{R}^r$, Ω is a connected open subset of $\mathbf{R}^n$, the set M_0 is compact in $\mathbf{R}^n$, $M_0 \subset \Omega$, and "overline" stands for the closure of a set.

The standing assumptions are as follows:
1) U is a compact subset of $\mathbf{R}^r$.
2) $f(t,x,u)$ is continuous and continuously differentiable in all variables, and satisfies Lipschitz and linear growth conditions with respect to x.
3) $F(t,x,\rho)$, $\varphi(x,\rho)$, and $\rho_0(x)$ are nonnegative, continuously differentiable functions.

By admissible controls we mean smooth functions $u(\cdot)$ with values in U, i.e., $u(\cdot) \in C^1(T_0, \mathbf{R}^r)$, $u(t) \in U$, $t \in T_0$. Given an admissible control function $u(\cdot)$, consider a bundle $\cup_{x_0 \in M_0} x(\cdot, x_0; u)$ of solutions to equation (1), starting from the set M_0. The set $M_{t,u}$ is defined as the section of the bundle at a given t, i.e.,

$$M_{t,u} = \{x_t = x(t,x_0;u) : x(\cdot,x_0;u) \text{ is a solution to (1), (3) under control } u(\cdot),\ x_0 \in M_0\}.$$

Problem (1)–(6) consists in optimal control of trajectory bundles considering the density of distribution. Previously, in [2], modeling issues and analysis of dynamics of trajectory bundles were addressed, and necessary conditions for optimality were given. Note that, in a number of practical applications, controls are smooth. In this work, by using technique [1], we propose necessary conditions for optimality in the class of smooth control functions.

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SOME DIFFICULTIES OF NUMERICAL SOLUTION OF OPTIMIZATION PROBLEMS WITH A BILLIONS OF VARIABLES [1]

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Optimization problems with large and very large dimensions ("Huge Scale optimization problems") occur naturally in a wide range of scientific fields - images recognition, machine learning, big data analysis, optimization of atomic and molecular clusters, analysis of genomic chains, analysis of telecommunication networks and many others. Yu.E. Nesterov recently proposed (see [1]) the following classification of optimization problems by number of optimized variables: "Small" - up to 100 variables, "Medium" - from 10^3 to 10^4, "Large" - from 10^5 to 10^7 and "Huge" - more than 10^8 variables. Steady progress of modern computer technology, especially in parallel architectures, and its availability expansion to a wide user gives optimism in investigation of solving optimization problems with discussed dimensions. "Bottleneck" in this scientific topics in our opinion is algorithms and optimization computational technologies weakness. It can be argued that the statement of the problem, "social inquiry" and technical probability are ripe, the next "move" should do mathematics and mathematicians.

Vigorous research of discussed problem conducted in a number of scientific organizations, both in Russia and abroad. In Belgium, serious progress on huge problems achieved by a team of Yu.E. Nesterov, in U.S. successfully working group of A.I. Nemirovsky and several others, in UK – P. Richtarik group, in Russia - group of A.V. Gasnikov in MIPT (PreMoLab). The number of scientific publications on this topic is growing rapidly.

The report discusses the proposed algorithms for solving test and applied multidimensional optimization problems. The possibility of modifying the method proposed for about half a century ago by B.T. Polyak in [2] is investigated. With using algorithms based on this approach, it was possible to solve a number of separable convex optimization problems with dimension up to 10^{11}. With the use of previously implemented techniques (see, eg. [3]) authors developed specialized computational technologies for Keating potential optimization problems with number of variables more than 10^7. Also was solved a number of Morse potential optimization for atomic-molecular clusters with world-record sizes. The results of numerical experiments were obtained using both personal computers and high-performance computing systems.

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LINEAR-QUADRATIC OPTIMAL CONTROL PROBLEM: SADDLE-POINT APPROACH[1]

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In the Hilbert space we consider the linear-quadratic optimal control problem with fixed left (x_0) and moving right (x_1) ends of trajectory $x(\cdot)$:

$$(x_1^*, x^*(\cdot), u^*(\cdot)) \in \mathrm{Argmin}\left\{ \langle Sx_1^*, x_1\rangle + \int_{t_0}^{t_1} (\langle Q_1(t)x^*(t), x(t)\rangle + \langle Q_2(t)u^*(t), u(t)\rangle)dt \ \mid \right. \quad (1)$$

$$A_1 x_1 \le a_1,\ x_1 \in \mathrm{R}^n, \quad (2)$$

$$\frac{d}{dt}x(t) = D(t)x(t) + B(t)u(t), \quad x(t_0) = x_0,\ x(t_1) = x_1^*, \quad (3)$$

$$x(\cdot) \in \mathrm{AC}^n[t_0, t_1],\ u(\cdot) \in \mathrm{U}\}, \quad (4)$$

$$\mathrm{U} = \{u(\cdot) \in \mathrm{L}_2^r[t_0, t_1] |\ \frac{1}{2}\|u(\cdot)\|^2_{L_2^r} \le \mathrm{C}^2\}. \quad (5)$$

More specifically:

— objective functional (1) is the sum of the terminal and integral components of the quadratic form;

— optimization problem (1) is solved with additional terminal constraints (2);

— controlled dynamics of the system is linear (3);

— phase trajectories $x(\cdot)$ are assumed to be continuous functions of the class of absolutely continuous functions $\mathrm{AC}^n[t_0, t_1]$ (4);

— controls $u(\cdot)$ are bounded by the norm $\mathrm{L}_2^r[t_0, t_1]$, i.e., integrally bounded (5);

— Reachability set is the whole space R^n or subspace.

We need to find a control $u^*(\cdot) \in \mathrm{U}$ such that the corresponding trajectory $x^*(\cdot) \in \mathrm{AC}^n[t_0, t_1]$ should connect the starting point x_0 with the minimum point x_1^* of the objective functional at the right end. Similar formulations of the problem and approaches to solving problems were considered in [1], [2].

Unlike the traditional approach, we treat here the optimal control problem not as a problem of optimization, but as a saddle-point problem. The required solution of the problem is a saddle point of the Lagrangian with components such as control, phase and conjugate trajectories, and terminal variables.

To solve the problem we suggest the saddle-point iteration method. Convergence of the method to the solution is proved.

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NUMERICAL METHOD FOR OPTIMAL CONTROL PROBLEMS GOVERNED BY NONLINEAR HYPERBOLIC SYSTEMS OF PDES

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We present two numerical methods for solving tracking-type optimal control problems subject to hyperbolic conservation and balance laws in one and two space dimensions. Our approach is based on the formal optimality system and requires numerical solutions of the hyperbolic system of conservation or balance laws forward in time and a corresponding nonconservative linear adjoint system backward in time.

We use a second-order shock-capturing finite volume method for the forward problem, our particular choice is the Godunov-type central-upwind scheme originally developed for general multidimensional hyperbolic systems of conservation laws. For the backward problem we explore two different strategies. In the scalar case, we use a highly accurate Lagrangian discrete characteristics method. In the system case, we develop a second-order Roe-type upwind finite volume scheme.

We illustrate the performance of the proposed numerical methods on a number of optimization problems constrained by both linear and nonlinear scalar conservation laws and a duct design problem as well as the Euler equations of gas dynamics or isothermal gas dynamics equations. Both smooth and discontinuous prescribed terminal states are considered.

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ON PROBLEM OF CHOOSING CONVENIENT REDUCTION OF WAVE EQUATION TO EQUIVALENT FIRST ORDER SYSTEMS IN OPTIMAL CONTROL PROBLEMS[1]

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There exist two approaches in studying controlled processes described by PDE of second and higher orders. The first approach is to deal with the optimal problem in terms of the existing equation. The second is to reduce the initial equation to a system of first order equations. In problems of optimal control of ordinary differential equations the second approach is far more popular. However, in optimal control of partial differential equations historically the first approach dominates. In multiple papers dedicated to Goursat-Darboux control problem it turned out that using the first approach the associated conjugate system can be written only in integral form. This fact makes it much more difficult to use this conjugate system in general study and numerical methods construction for the control problem.

The advantages of using the second approach were made clear in study of control problem of wave equation. Its reduction to a system of first order equations enabled us to introduce a clear and convenient concept of a generalized solution [1], to obtain the necessary optimality conditions [2], and numerical methods [3] for the stated problem.

The aim of the report is to generalize and develop further this approach.

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NUMERICAL METHODS FOR OPTIMAL CONTROL PROBLEM OF PARABOLIC EQUATION[1]

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We consider an optimal control problem of parabolic equation. Necessary optimality conditions are given for the optimal control problem. We construct methods of improvement of admissible control functions based on the optimality conditions. The numerical experiment is carried out.

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OPTIMALITY CONDITIONS FOR IMPULSIVE CONTROL PROBLEMS WITH INTERMEDIATE STATE CONSTRAINTS[1]

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This report deals with impulsive optimal control problems with trajectories of bounded variation and impulsive controls (regular vector measures). The problem under consideration is characterized by two main features. First, the dynamical control system is linear in the impulsive control and does not possess the so-called well-posedness property. Second, there are intermediate state constraints on the one-sided limits of the trajectory of bounded variation.

An impulsive control system of the following form

$$dx(t) = f\big(t, x(t), u(t)\big)dt + G\big(t, x(t)\big)\pi(\mu), \quad u(t) \in U \text{ a.e. on } T, \quad \pi(\mu) \in \mathcal{W}(T, K) \tag{1}$$

is considered. Here, $T = [a, b]$ is a time interval, U is a compact set in R^r, K is a convex closed cone in R^m, $u(\cdot)$ and $\pi(\mu)$ are an ordinary control and an impulsive control correspondingly, and $x(\cdot) \in BV\,(T, R^n)$, where $BV\big(T, R^n\big)$ is the Banach space of R^n-valued functions of bounded variation on T. The set of the impulsive controls $\mathcal{W}(T, K)$ consists of all pairs $\big(\mu, \gamma(\mu)\big)\big\}$ such that μ is a K-valued bounded Borel measure on T and $\gamma(\mu)$ is the set $\{d_s, \omega_s(\cdot)\}_{s\in S}$ satisfying the following conditions:

(a) $S \supseteq S_d(\mu) := \{s \in T \mid \mu(\{s\}) \neq 0\}$, S is at most denumerable set in T;

(b) $\forall\ s \in S \quad d_s \geq 0,\ \ \omega_s : [0, d_s] \to co\ K_1,\ \ d_s \geq ||\mu\big(\{s\}\big)||,\ \ \displaystyle\int_0^{d_s} \omega_s(\tau)d\tau = \mu\big(\{s\}\big)$;

(c) $\displaystyle\sum_{s\in S} d_s < \infty.$

Here, $K_1 = \{v \in K \mid ||v|| = 1\}$ and $||v|| = \sum_{j=1}^m |v_j|$. The solution concept of (1) we adopt with some modifications was stated in [1].

In this report, necessary and sufficient optimality conditions that correspond to the Hamilton–Jacobi canonical optimality theory [2, 3] and involve some sets of Lyapunov type functions are presented and discussed. These functions are strongly or weakly monotone solutions of the corresponding Hamilton-Jacobi inequalities and may be compound, i.e. defined piecewise in the variable t. Some examples illustrating the optimality conditions are discussed.

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ON FEEDBACK NECESSARY OPTIMALITY CONDITIONS FOR LINEAR IMPULSIVE CONTROL PROBLEMS [1]

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The talk is concerned with some applications of modern techniques of weakly monotone solutions to Hamilton-Jacobi inequalities for a class of nonconvex optimal impulsive control problems with trajectories of bounded variation. The system's dynamics is described by a measure differential equation, which is linear with respect to a state variable. We discuss nonstandard duality and nonlocal variational necessary optimality conditions with feedback descent controls [1].

Consider the following optimal control problem (P): Minimize $I = \langle c, x(1)\rangle$ subject to the dynamical system

$$dx = [A(t,u)x + a(t,u)]\ dt + [B(t,u)x + b(t,u)]\ \vartheta(dt), \quad x(0-) = x_0.$$

Here, $x(t) \in R^n$, usual control u is a Borel measurable function with values in a given compact set $U \subset R^r$, and ϑ is impulsive control in the sense [2], which is a certain family of Borel measures and measurable functions. The main component of ϑ (the actual impulsive control) is a scalar Borel measure μ. The system is subject to an additional constraint on the total impulse of control: $\nu([0,T]) \leq M$ with a given $M > 0$, where a scalar nonnegative measure ν (another component of ϑ) is a majorant of the total variation of μ.

For nonconvex problem (P) we establish the nonstandard duality: We prove that (P) is an equivalent of a certain comparison problem, which is formulated in terms of adjoint trajectories from the Maximum Principle [3]. Admissible sets of these problems, their minimizing sequences and values are shown to coincide. By means of the well-known time reparameterization technique [3], problem (P) is transformed to an equivalent conventional optimal control problem (RP) with absolutely continuous trajectories. In the reduced problem, we face a terminal constraint imposed on the component of state trajectory, that corresponds to time of the original system. For problem (RP) we obtain variational necessary optimality conditions within Pontryagin formalizm, but with the use of feedback controls [1]. Unfortunately, the discussed result is not yet completely translated into the original impulsive framework. Still, it is effectively used for design of numerical algorithms for optimal impulsive control.

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ON GLOBAL OPTIMALITY CONDITIONS IN OPTIMAL CONTROL[1]

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Consider the standard control system (for example, see [2]):

$$\left.\begin{array}{c} \dot{x}(t) = f(x(t), u(t), t) \quad x(t_0) = x_0, \quad t \in T = [t_0, t_1], \\ u(\cdot) \in \mathcal{U} = \{\, u(\cdot) \in L^r_\infty(T) \mid u(t) \in U \quad \overset{\circ}{\forall}\, t \in T \,\}, \end{array}\right\} \tag{1}$$

and the optimal control problem

$$J_0(u) \downarrow \min_u, \quad u \in \mathcal{U}, \quad J_i(u) \le 0, \quad i = 1, \ldots, m \tag{$\mathcal{P}$}$$

with Bolza functional as follows:

$$J_i(u) := F_{i1}(x(t_1, u)) + \int\limits_T F_i(x(t, u), u(t), t)\, dt, \quad i \in \{\, 0, 1, \ldots, m \,\}. \tag{2}$$

In (2) the functions $F_{i1}(x)$ and $F_i(x, u, t)$ are the d.c. functions w.r.t. the state, i.e.

$$\left.\begin{array}{c} F_{i1}(x) = g_{i1}(x) - h_{i1}(x), \\ F_i(x, u, t) = g_i(x, u, t) - h_i(x, t), \quad t \in T, \end{array}\right\} \tag{3}$$

where the functions $x \mapsto g_{i1}(x)$, $x \mapsto h_{i1}(x)$ are convex on $I\!R^n$, while the functions $g_i(x, u, t)$, $h(x, t)$ are convex w.r.t. the variable $x \in I\!R^n$.

For nonconvex problems of type ($\mathcal{P}$) we propose two types Global Optimality Conditions (GOC)[3] connected with the maximum principle of Pontryagin [1]. On the base of GOC we developed local and global Search Methods and investigated its convergence [4]. In addition we performed the testing the methods which demonstrated the effectiveness of the developed approach on a rather large field of test-problems of a big size.

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ON NUMERICAL METHOD FOR GLOBAL SEARCH IN NONCONVEX OPTIMAL CONTROL PROBLEMS[1]

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Consider the following linear control system:

$$\dot{x}(t) = A(t)x(t) + b(u(t), t) \quad \overset{\circ}{\forall} t \in T = [t_0, t_1], \quad x(t_0) = x_0, \tag{1}$$

$$u(\cdot) \in \mathcal{U} = \{\, u(\cdot) \in L^r_\infty(T) \mid u(t) \in U \quad \overset{\circ}{\forall} t \in T \,\}, \tag{2}$$

where $A(\cdot)$ is $(n \times n)$-matrix function with continuous elements $t \mapsto a_{ij}(t)$, $i, j = 1, 2, \ldots, n$ on $T := [t_0, t_1]$, and U is a compact set. Assume also that vector function $(u, t) \mapsto b(u, t)$ is continuous with respect to variables $u \in I\!R^r$, and $t \in T$.

We study the following optimal control (OC) problem:

$$(\mathcal{P}): \qquad J(u) = F_1(x(t_1, u)) + \int\limits_T F(x(t, u), t)\, dt \downarrow \min_u, \quad u(\cdot) \in \mathcal{U}. \tag{3}$$

Functions $x \mapsto F_1(x)\colon I\!R^n \to I\!R$ and $(x, t) \mapsto F(x, t)$ (see (3)) are d.c. functions (A.D. Alexandrov's functions, see [1]), therefore these function can be represented as a difference of two convex functions with respect to variable x (for all $t \in T$):

$$F_1(x) = g_1(x) - h_1(x), \qquad F(x, t) = g(x, t) - h(x, t) \quad \forall x \in \Omega \subset I\!R^n, \quad t \in T, \tag{4}$$

where $x \mapsto g_1(x)$, $x \mapsto h_1(x)$, $x \mapsto g(x, t)$, and $x \mapsto h(x, t)$ are convex functions with respect to variable x for all $t \in T$.

Note that the OC problem $(\mathcal{P})$ is nonconvex (см. [1,2]), and it might possess a number processes, satisfying the Pontryagin maximum principle (PMP), which are rather far from a global solution. This nonconvexity is created by objective functional $J(\cdot)$ of problem $(\mathcal{P})$.

On the basis of the global optimality conditions [1] we propose the method for searching globally optimal processes in problem $(\mathcal{P})$, which combines a local search search procedure and the procedure for improving process, satisfying PMP [2]. Under certain assumptions we prove the theorem of convergence of the global search method. Also we performed a numerical experiment, which showed operability and efficiency of the developed method on a series of test problems with quadratic nonconvex objective functional.

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OPTIMIZATION ALGORITHM FOR NON-CONVEX RELAY OPTIMAL CONTROL PROBLEMS[1]

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We consider an optimal control problem (OCP) of relay type in following statement

$$\dot{x} = f(x, u, t), x(t_0) = x^0, t \in T = [t_0, t_1] \tag{1}$$

$$u \in U = [u_j \in \underline{u}_j; \overline{u}_j], j = \overline{1, r} \tag{2}$$

$$I(u) = \varphi(x(t_1)) \to min. \tag{3}$$

Vector-function $f(x, u, t)$ and scalar function $\varphi(x)$ are assumed to be continuously differentiable in all arguments except t.

Modification algorithms are developed for researching non-convex relay OCP (1)—(3) which based on consideration of the specific structure of the control. We use elements of the theory of machine learning (it is implemented an approximation by the Shepard's function [1 , 2] for the data of the objective functional values for different stochastic controls) and the genetic search (it is included the original crossover and mutation mechanisms for constructing relay controls [3]). We have carried out a comparison of the proposed approaches to the method multistart oriented to researching multiextremal OCP [4] and tunneling method for global search which based on constructing of curves on the attainability set to exit the local extremum [5]. The testing results allow us to demonstrate the effectiveness of the developed algorithms for relay optimal control problems.

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OPTIMIZATION IN INVERSE PROBLEMS

MATHEMATICAL MODELLING OF HEAT TRANSFER IN PACKED BEDS

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In the report we model three problems. The first one is the "homogeneous" flow model [1], the simplest approach to the problem, the two phase flow is assumed to be a single phase flow having pseudo properties arrived at by suitably weighting the properties of the individual phases. The main difficulty in particular problem is to solve Dirichlet boundary problem for Poisson equation

$$\Delta u(x,y) = f(x,y), \;\; (x,y) \in D,$$

∂D is boundaries of the domain D with the boundary conditions on $\partial D : u(x,y)|_{\partial D} = c(x,y)$. The multiply connected domain D and interior separation lines are the main challenges [2, p.167].

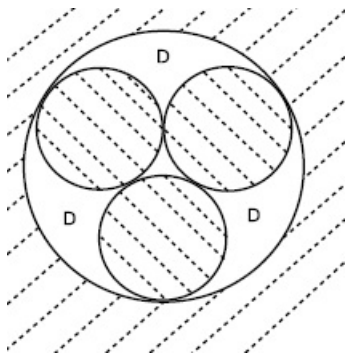

Рис. 1: Packed beds: example of the domain D

The second problem concerns the "separated" flow model. In this approach the two phases of the flow are considered to be artificially segregated. Two sets of basic equations can now be written, one for each phase. The information must be forthcoming about the flow area occupied by each phase and about various kinds of interactions at the interface. This information is integrated into the basic equations on the basis of simplified models of the flow. The third one addresses the "flow pattern" models. In this more sophisticated approach the two phases are considered to be arranged in one of several prescribed geometries. These geometries are based on the various configuration of flow patterns found when both the phases are flowing together. The basic equations are solved within the framework of each of these idealized representations. In order to apply these models it is necessary to know when each should be used and to be able to predict the transition from one pattern to another.

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WEIGHTED MATRIX STRUCTURAL ADJUSTMENT IMPROPER LINEAR PROGRAMMING PROBLEMS OF THE 1ST KIND[1]

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The report, in the continuation of research initiated in [1], the problem of P matrix correction of improper linear programming (LP) of the 1st kind of the minimum weighted Euclidean norm in the case where the elements of the augmented matrix $[A \ -b]$ coefficients of its limitations divided into a plurality of adjustable $[K^+ \ k^+]$ and uncorrectable $[K^- \ k^-]$ elements.

Theorem. *If the problem P has a solution, then the desired vectors and matrices can be constructed by direct formulas that depend on the solutions (soluble under these conditions) unconstrained minimization problem.*

$$\Phi(\tilde{x}) = \sum_{i=1}^{m} \frac{(b - A\,(\mathrm{diag}(\tilde{x})\tilde{x}))_i^2 \cdot \left| s\left(\begin{bmatrix} \mathrm{diag}(\tilde{x})\tilde{x} \\ 1 \end{bmatrix}, \begin{bmatrix} \mathcal{H}_{i*}^\top \\ \chi_i \end{bmatrix}\right) \cdot \mathrm{diag}\left(\begin{bmatrix} \mathcal{W}_{i*}^\top \\ \omega_i \end{bmatrix}\right)\right|^2}{\left(s^\top\left(\begin{bmatrix} \mathrm{diag}(\tilde{x})\tilde{x} \\ 1 \end{bmatrix}, \begin{bmatrix} \mathcal{H}_{i*}^\top \\ \chi_i \end{bmatrix}\right) \cdot s\left(\begin{bmatrix} \mathrm{diag}(\tilde{x})\tilde{x} \\ 1 \end{bmatrix}, \begin{bmatrix} \mathcal{H}_{i*}^\top \\ \chi_i \end{bmatrix}\right)\right)^2} \to \min,$$

where the coefficients $\begin{bmatrix} \mathcal{W}_{i*}^\top \\ \omega_i \end{bmatrix}$ *determine the weight of each i-th row correction matrix $[H \ -h]$ in the objective function, $\mathcal{H}_{i*}$ - i-th row of the matrix $\mathcal{H}$,* $s(p,q) = (s(p,q)_i) \left| \begin{array}{ll} s(p,q)_i = p_i, & \texttt{if } q_i \neq 0 \\ s(p,q)_i = 0, & \texttt{if } q_i = 0 \end{array} \right.$, $\mathcal{H} = (\mathcal{H}_{ij}) \left| \begin{array}{ll} \mathcal{H}_{i,j} = 1, & \texttt{if } \{i,j\} \in \mathbf{K}^+ \\ \mathcal{H}_{i,j} = 0, & \texttt{if } \{i,j\} \in \mathbf{K}^- \end{array} \right.$, $\chi = (\chi_i) \left| \begin{array}{ll} \chi_i = 1, & \texttt{if } \{i\} \in \mathbf{k}^+ \\ \chi_i = 0, & \texttt{if } \{i\} \in \mathbf{k}^- \end{array} \right.$.

Analytical formulas for the gradient of this function are given [2]. Computational procedure of minimization with the use of an algorithm Broyden-Fletcher-Goldfarb-Shanno is implemented. Results of model problems of middle dimension with sparse matrix coefficients from the catalog "lp/infeas"storage Netlib, illustrating the convergence of the argument and the objective function and the distribution of elements on the amendments are given.

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THE REGULARIZATION OF THE PROBLEM OF CONSTRUCTION OF CAPITAL PRODUCTION FUNCTIONS ON INVESTMENT DATA [1]

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A method for the construction of standard production functions (PFs), which have effective capital (EC) as one of factors, was suggested in [1], for the case when production statistics contains investment data instead of capital ones. Estimation of a PF's parameters uses a capital dynamic equation defined by investments as well as a depreciation and coefficient of investment mastering. The PF's parameters and ones of dynamic equation are being estimated with an initial value of EC simultaneously. The functions are being constructed in subsequently complicating classes starting from the simplest Cobb and Douglass one. In general case, the new Least Square estimation problem of the extended parameters collection is an ill-conditioned nonlinear programming problem which demands usage of effective optimization techniques and a regularization on the base of additional information. In the referred paper a special variant of the continuation method [2] was suggested which can overcome complexity of nonlinear minimization.

The work [3] develops the approach of [1] in the next respects. A coefficient of the realizability of investments is introduced in the dynamics equation. This coefficient represents a ratio of really used capital investments which is less of one because of corruption. As an additional means for overcoming of computational complexities the transform to the index form of PFs is used [4]. Also a regularization method for the complex parameter estimation is suggested, which uses an expert information about the dynamics factors and a stabilization of the transition from an estimated PFs' class to a more complex one. Results of realization of the proposed model and techniques to real data for some regional Russian economies will be presented.

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RESTORING THE PARAMETERS OF CONJUGATED PAIRS OF LINEAR ALGEBRAIC EQUATION SYSTEMS BY A SET SOLUTION [1]

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The report observes the theorem of recovering the parameters of a conjugated pair of linear algebraic equation systems by a set solution using an interval criterion.

Theorem. *The* $A \in \mathbf{R}^{m\times n}$ *family of matrices and the* $b \in \mathbf{R}^m$, $c \in \mathbf{R}^n$, *families of vectors that guarantee that the set* $\bar{x} \in \mathbf{R}^n$ *and* $\bar{u} \in \mathbf{R}^m$ *vectors belong to the*

$$\begin{cases} Ax = b, \\ u^\top A = c^\top, \end{cases}$$

set of solutions of a conjugated pair of systems of linear algebraic equations, and at the same time, $\|A\| \le \alpha,\ \|b\| \le \beta,\ \|c\| \le \gamma$, *where* $\alpha > 0,\ \beta > 0,\ \gamma > 0$ *can be constructed using*

$$b = \lambda \frac{\bar{u}}{\bar{u}^\top \bar{u}} + \lambda \left(I_m - \frac{\bar{u}\bar{u}^\top}{\bar{u}^\top \bar{u}} \right) \Delta b,\ c = \lambda \frac{\bar{x}}{\bar{x}^\top \bar{x}} + \lambda \left(I_n - \frac{\bar{x}\bar{x}^\top}{\bar{x}^\top \bar{x}} \right) \Delta c,\ A = \frac{1}{\lambda} b c^\top,$$

formulas, where $\|\cdot\|$ *stands for, depending on the content, the Euclidean matrix or vector norm, the scalar parameter* λ *is calculated using the*

$$\lambda \le \bar{\lambda} = \min \left(\frac{\alpha}{\bar{\alpha}}, \frac{\beta}{\bar{\beta}}, \frac{\gamma}{\bar{\gamma}} \right),$$

rule,

$$\bar{\beta} = \sqrt{\frac{1}{\bar{u}^\top \bar{u}} + \Delta b^\top \left(I_m - \frac{\bar{u}\bar{u}^\top}{\bar{u}^\top \bar{u}} \right) \Delta b}, \quad \bar{\gamma} = \sqrt{\frac{1}{\bar{x}^\top \bar{x}} + \Delta c^\top \left(I_n - \frac{\bar{x}\bar{x}^\top}{\bar{x}^\top \bar{x}} \right) \Delta c}, \quad \bar{\alpha} = \bar{\beta} \cdot \bar{\gamma},$$

$\Delta b \in \mathbf{R}^m$, $\Delta c \in \mathbf{R}^n$ *are random vectors,* I_m, I_n *are singular matrices of size* m *and* n, *accordingly.*

At the same time $\|A\| = \lambda \cdot \bar{\alpha},\ \|b\| = \lambda \cdot \bar{\beta},\ \|c\| = \lambda \cdot \bar{\gamma}$.

The report ends with a numerical experiment with a model example.

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NUMERICAL SOLUTION OF WEAKLY REGULAR VOLTERRA INTEGRAL EQUATIONS OF THE 1ST KIND

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We propose new numerical method for solution of the following linear scalar Volterra integral equation of the 1st kind

$$\int_0^t K(t,s)x(s)ds = f(t), \;\; 0 \le s \le t \le T, \;\; f(0) = 0,$$

where kernel is defined as follows

$$K(t,s) := \begin{cases} K_1(t,s), \; t,s \in m_1, \\ \dots \;\; \dots\dots\dots \\ K_n(t,s), \; t,s \in m_n, \end{cases} \quad \begin{array}{l} m_i := \{t,s \mid \alpha_{i-1}(t) < s < \alpha_i(t)\}, \\ \alpha_0(t) = 0, \; \alpha_n(t) = t, \; i = \overline{1,n}, \end{array}$$

$\alpha_i(t)$, $f(t) \in \mathcal{C}^1_{[0,T]}$, $K_i(t,s)$ have continuous derivatives (w.r.t. t) for $t,s \in \overline{m_i}$, $K_n(t,t) \neq 0$, $\alpha_i(0) = 0$, $\;0 < \alpha_1(t) < \alpha_2(t) < \dots < \alpha_{n-1}(t) < t$, $\alpha_1(t), \dots, \alpha_{n-1}(t)$ increase at least in the small neighborhood $0 \le t \le \tau$, $0 < \alpha_1'(0) \le \dots \le \alpha_{n-1}'(0) < 1$. The mid-rectangular quadrature rule is employed for the numerical method construction. The accuracy of proposed numerical method is $\mathcal{O}(1/N)$. We introduce the following mesh $\Omega_x^N := \{t_i | t_i = i/N, i = 0, \dots, N\}$, the mesh can be non-uniform: $0 = t_0 < t_1 < t_2 < \dots < t_N = T$, $\;h = \max\limits_{i=\overline{1,N}}(t_i - t_{i-1}) = \mathcal{O}(N^{-1})$ and seek the approximate solution $x_N(t) = \sum\limits_{i=1}^N x_i \delta_i(t)$, $t \in (0,T]$, $\delta_i(t) = \begin{cases} 1, & \text{for } t \in \Delta_i = (t_{i-1}, t_i] \\ 0, & \text{for } t \notin \Delta_i \end{cases}$, where the coefficients x_i, $i = \overline{1,N}$ are under determination. Let us consider the equation $\int\limits_0^{t/3} (1 + t - s)x(s)\,ds - \int\limits_{t/3}^t x(s)\,ds = \frac{t^4}{108} - \frac{25t^3}{81}$, $t \in [0, 2]$, where $\bar{x}(t) = t^2$ is exact solution. Table below demonstrates the errors $\varepsilon_N = ||x^N(t_i) - \bar{x}(t_i)||_{\Omega^N}$ and order of convergence $p^N = \log_2 \frac{D^N}{D^{2N}}$ based on maximum pointwise two-mesh differences $D^N = ||x^N(t_i) - \bar{x}^{2N}(t_i)||_{\Omega^N}$ without *a priori* knowledge of exact solition. This is joint work

	32	64	128	256	512	1024	2048	4096
ε_N	0.13034	0.07804	0.03989	0.01975	0.01002	0.00508	0.00256	0.00128
D^N	0.07462	0.03815	0.02013	0.00975	0.00514	0.00259	0.00129	0.00065
p^N	0.96774	0.92207	1.04619	0.92217	0.98864	1.00716	0.98639	1.00198

with Denis Sidorov and Alexander Tynda.

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AUTOMATED ANTHROPOMETRIC MEASUREMENTS AND FEATURES EXTRACTION FROM 2D IMAGES

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Our objective is real-time human body attributes recognition (height, shape, hair/eye/skin color, clothes color and gender) from video/pictures taken in a controlled (light and background) environment. In particular we measure the length of body, length of arms, width of shoulder, chest and wrist circumference. The detection of the contour of body jointly with calibration theory and background subtraction algorithms is used. In fact, images have noises so detection contour faces some issues we overcome using regularization theory. We also use background subtraction algorithm to detect change before and after of scene when object appears. It supports exactly detect contour.

Our purpose is build robust measure software for normal camera and mobile application. The applications area involve many fields including security.

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ON THE ORDER OF SINGULARITY OF THE GENERALIZED SOLUTION OF THE VOLTERRA INTEGRAL EQUATION OF CONVOLUTIONAL TYPE IN BANACH SPACES[1]

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Let us denote E_1, E_2 are real Banach spaces, $\mathbf{u} = u(t)$, $\mathbf{f} = f(t)$ are functions of nonnegative real argument t and with values in E_1 and E_2 respectively. Let us consider integral equation

$$B\mathbf{u} - g * A\mathbf{u} = \mathbf{f}. \tag{1}$$

Here B and A are closed linear operators from E_1 to E_2 at that $\overline{D(B)} = \overline{D(A)} = E_1$ and $D(B) \subseteq D(A)$, kernel $g(t)$ is numerical function. It is assumed that the operator B is Fredholm, i. e. $\overline{R(B)} = R(B)$ and $\dim N(B) = \dim N(B^*) = n < +\infty$, and the function $g(t)$ has a null of order r at $t = 0$.

For example, next boundary value problem can be reduced to this integral equation

$$(\Delta - \alpha)\varphi(t, \bar{x}) + \int\limits_0^t (t - \tau)\beta\varphi_{x_N^2}(\tau, \bar{x})d\tau = f(t, \bar{x}),\ t > 0,\ \bar{x} = (x_1, \ldots, x_N) \in \Omega;\ \varphi(t, \bar{x})|_{\bar{x}\in\partial\Omega} = 0,$$

where Ω is domain of $\mathbb{R}^N$ with boundary $\partial\Omega$ of C^∞ class, constants α and β are not equal to zero. In the case of $N = 3$, this boundary value problem describes the low-frequency electronic (ionic) magneto-acoustic oscillations in an external magnetic field [1].

Using the method of fundamental operator-function of degenerated integro-differential operator in Banach spaces [2] we have constructed generalized solution of integral equation (1) in the class of distributions with left-bounded support proved its uniqueness. We have shown the connectivity between order of generalized solution singularity [3] and order r of function $g(t)$ null at point $t = 0$. Also we have obtained the conditions at which the order of singularity of the generalized solution is equal to zero, it means that generalized solution coincides with the continuous (classical) solution of integral equation (1). Abstract results are illustrated by the example of boundary value problem in plasma physics.

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SOLUTION OF LINEAR AND NONLINEAR VOLTERRA INTEGRAL EQUATIONS OF THE FIRST KIND WITH PIECEWISE SMOOTH KERNELS

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Volterra integral equations of the first kind $\int\limits_0^t K(t,s)x(s)ds = f(t),\ f(0)=0,\ 0 \le s \le t \le T$, with piecewise smooth kernels

$$K(t,s) := \begin{cases} K_1(t,s),\ t,s \in m_1, \\ \dots \quad \dots\dots\dots \\ K_n(t,s),\ t,s \in m_n, \end{cases} \qquad \begin{array}{l} m_i := \{t,s \mid \alpha_{i-1}(t) < s < \alpha_i(t)\}, \\ \alpha_0(t)=0,\ \alpha_n(t)=t,\ i=\overline{1,n} \end{array}$$

are considered. Here $\alpha_i(t)$, $f(t) \in \mathcal{C}^1_{[0,T]}$, $K_i(t,s)$ have continuous derivatives (w.r.t. t) for $t,s \in \overline{m_i}$, $K_n(t,t) \ne 0$, $\alpha_i(0)=0$, $\quad 0 < \alpha_1(t) < \alpha_2(t) < \dots < \alpha_{n-1}(t) < t$, $\alpha_1(t),\dots,\alpha_{n-1}(t)$ increase at least in the small neighborhood $0 \le t \le \tau$, $0 < \alpha_1'(0) \le \dots \le \alpha_{n-1}'(0) < 1$. The following results will be reported for this equation: condition for the existence of a unique solution; conditions for the existence of parametric solutions. Based on these results the approximate methods (successive approximations and asymptotic methods) are constructed. Taking into account obtained conditions for the existence of a unique solution, the numerical methods are proposed in [4,5] and their first order convergence is observed. The theoretical results [1-3] are generalized on the cases of matrix kernels in [6] and operator coefficients $K_i(t,s)$ in [7]. The proposed theory can be employed for solutions of the problems in power systems development modeling, including optimal equipment replacement [5,8]. New results concering the generalizations on nonlinear case and piecewise smooth source function will be also reported.

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ON UNBOUNDED SOLUTIONS OF AN INITIAL VALUE PROBLEM

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Consider the differential equation

$$B(t)\frac{dx}{dt} = A(t)x(t) + f(t) \tag{1}$$

with the initial condition

$$\lim_{t\to 0} B(t)x(t) = y_0, \tag{2}$$

$B(t)$, $A(t)$, $f(t)$ - analytic in a neighborhood of zero . It is assumed that $B(0)-$ Fredholm operator, $\{\phi_1, \dots, \phi_n\}$ - a basis of Ker$B(0)$, $\{\psi_1, \dots, \psi_n\}$ - a basis of Coker$B(0)$.

$$\mathrm{Ker}B(0) \subseteq \bigcap_{i=0}^{k-1} \mathrm{Ker}B^{(i)}(0), \quad \det[\langle B^{(k)}_{(0)}\phi_i, \psi_j\rangle]_{i,j=\overline{1,n}} \neq 0,$$

$$\det[\lambda\langle B^{(k)}_{(0)}\phi_i, \psi_j\rangle - \langle A^{(k-1)}_{(0)}\phi_i, \psi_j\rangle]_{i,j=\overline{1,n}} \neq 0$$

for $\lambda = -k+1, -k+2, \dots$.
If $k \geq 2$, then the additional assumption that

$$\mathrm{Ker}B(0) \subseteq \bigcap_{i=0}^{k-2} \mathrm{Ker}A^{(i)}(0), \ , \ , \ , \ , \det[\langle A^{(k-1)}_{(0)}\phi_i, \psi_j\rangle]_{i,j=\overline{1,n}} \neq 0$$

Then we have

Theorem. *Initial value problem* (1), (2) *in a punctured neighborhood* $0 < |t| < r$ *is in the class of analytic functions of a unique solution. Solution can be represented in the form of a Laurent series with a pole of order* $k-1$.

If $k = 1$, the point $t = 0$ is a removable singularity of solutions and we arrive at the well-known result (see , for example, [1]) .

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NUMERICAL METHOD FOR SOLVING AN INVERSE BOUNDARY VALUE PROBLEM OF HEAT CONDUCTION USING THE VOLTERRA EQUATIONS OF THE FIRST KIND[1]

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In the report I consider Volterra equation of the first kind

$$\int_0^t K_N(t-s)\,\phi(s)\,ds = g_\delta(t),\ 0 \le s \le t \le T, \tag{1}$$

where

$$K_N(t-s) = 2\pi^2 \sum_{p=1}^{N} (-1)^{p+1} p^2 e^{-\pi^2 p^2 (t-s)}, \tag{2}$$

which is reduced the solution of an inverse boundary value problem of heat conduction [1]. The right side of (1) is an approximation of function $g(t)$ for $\delta > 0$, so that $\forall\, t \ge 0$ we have $\|g_\delta(t) - g(t)\| \le \delta$.

To illustrate the specifics of Volterra integral equation of the first kind (1), (2) I present the numerical characteristics of the Volterra kernels $K_N \in C_\Delta$, $\Delta = \{t, s/0 \le s \le t \le T\}$ for fixed values N. The table contains the values K_N for $t = 0$, and the roots t^* of equations $K_N(t) = 0$, $N = \overline{10, 21}$, obtained with single precision.

N	t^*	$K_N(0)$	N	t^*	$K_N(0)$
10	0.01378	-1085.656	16	0.00913	-2684.532
11	0.01221	1302.788	17	0.00631	3020.099
12	0.01173	-1539.658	18	0.00809	-3375.405
13	0.01022	1796.268	19	0.00516	3750.449
14	0.01019	-2072.62	20	0.00735	-4145.234
15	0.00789	2368.705	21	0.00429	4559.757

I constructed an algorithms for the numerical solution of Volterra equation of the first kind (1), (2) based on the self-regularizing property of discretization procedure. Middle rectangles method and product integration method used as a "base". I found the parameters that define the discretization step. I performed a series of test calculations to test the effectiveness of the numerical method. Computing experiment showed that the difference methods converge on the grid step with the order $\mathcal{O}(h^2)$ in the absence of perturbations of the initial data.

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RECOVERING IMAGES REGISTERED BY DEVICE WITH INEXACT HARDWARE FUNCTION USING TIKHONOV'S REGULARIZED LEAST SQUARES METHOD [1]

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The using Tikhonov's regularized least squares method (RLS) [1] to solve approximate systems of linear algebraic equations (SLAE) are considered. Said method is applied to solve image recovery problem for image, registered by device with inexact hardware function.

RLS Problem [1, 2, 3]. Let an exact consistent finite-dimensional SLAE $A_0x = b_0$ exists, where $A_0 \in R^{m\times n}$, $b_0 \in R^m$, $x_0 \in R^n$ — normal solution of said SLAE. Instead of A_0 and b_0, approximate matrix A and vector b are set so that to meet the constraints $\|A_0 - A\| \leq \mu$, $\|b_0 - b\| \leq \delta < \|b\|$. It is required to find such $A_1 \in R^{m\times n}$, $b_1 \in R^m$, and $x_1 \in R^n$ that $\|A_1 - A\| \leq \mu$, $\|b_1 - b\| \leq \delta$, $A_1x_1 = b_1$, $\|x_1\| \to \min$.

The common method for solving discussed problem is to solve Euler regularized system $\left(A^TA + \alpha I\right) x_\alpha = A^Tb$ with regularization parameter $\alpha > 0$.

As it shown in [2], there are such conditions that leads to necessity of using negative regularization parameter in Tikhonov's regularized least squares method. This case is interesting from both a theoretical and practical point of view, because its computing implementation requires special tools to overcome the bad numerical conditioning.

It is known (see, e.g. [4]), that the image recovery problem (when image was registered by device with inexact hardware function) may be reduced to problem of solving approximate SLAE. And approximate SLAE may be solved by RLS for certain values of regularization parameter.

The values of regularization parameter which provide minimal error of image recovering were found in computational experiments. The specially designed algorithms with minimal numerical conditioning were used for image recovering in that experiments. Said algorithms are working with any values of regularization parameter including $\alpha < 0$.

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EQUILIBRIUM AND BILEVEL PROGRAMMING

ABOUT ONE BEHAVIOR MODEL OF GENERATING COMPANIES IN THE ELECTRICITY MARKET

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We consider the electricity market model of interaction between power producers (GenCo) and System Operator (SO), taking into account the limits on generation, capacities of lines, as well as transmission losses [1].

Power producers are able to affect the market price. The problem should take into account the oligopolistic situation in the market. A short description of the model is as follows. SO solves a generation scheduling problem, minimizing total costs of electricity generation and calculating the nodal prices (dual variables) on the basis of technical parameters of the power plants provided by the producers. Thus the oligopolistic situation in the market and GenCo's desire to receive a profit maximum is not considered. To increase their profit power producers deliberately distort real values of some technical parameters of the power plants thereby implicitly influencing the prices.

A mathematical formulation of the problem taking into account the factors of market environment is presented. The problem is considered in two-level statement. The upper level corresponds to the profit maximization of Generating Company with true cost functions.

The lower level of the problem corresponds to SO efforts to schedule generation and calculate local marginal prices (LMP) on the basis of total production cost minimization.

The two-level optimization problem can be interpreted as profit maximization for producer with modeling of following activity of SO for generation scheduling and nodal price determination.

The standard replacement of the lower level problem by the Karush-Kuhn-Tucker (KKT) optimality conditions is proposed. In order to validate the proposed approach and demonstrate its main features a numerical example is presented.

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STOCHASTIC COMPETITIVE FACILITY LOCATION PROBLEM WITH QUANTILE CRITERION [1]

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We suggest a statement of stochastic facility location problem as a discrete bilevel stochastic programming problem with quantile criterion [1]. There are two players (the leader and the follower) in the problem. They sequentially locate facilities to get the largest income from customers. The facility locations in a finite set of feasible points are leader's and followers's variables. The follower locates own facilities knowing a leader's variable. The leader takes into account an optimal value of the follower's variables to locate own facilities. If the optimal follower's variable is not unique, the leader takes into account a worst one from the own point-of-view, i.e. we consider the pessimistic position. Unlike problem [2], the demand for goods is assumed to be random. We suppose that the leader knows only the distribution of the random demand but the follower takes decision knowing the demand exactly. The quantile of the leaders's income (Value-at-Risk) is considered to be the objective function of the problem. The quantile is the income guaranteed with fixed probabilily.

In case of a discrete distribution of the random demand with finite number of realizations we suggest a method to reduce the problem to a deterministic bilevel programming problem using complementary binary variables.

We suggest a method to compute the value of the objective function for a fixed leader's varible and methods to compute lower and upper bounds of the optimal value of the objective function. We build an algorythm to search a suboptimal solution of the problem based on a local search with random initial points. The opportunity to search the exact solution is discussed.

Large numerical experiment is made to show the quality of the bounds of the optimal value of the objective function and accuracy of the solution obtained with the algorythm.

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ANALYSIS OF EXTRAPOLATION METHODS FOR SOLVING VARIATIONAL INEQUALITIES

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The report examines the problem of effective implementation of mathematical methods for solving variational inequalities. Mathematical modeling plays an important role in the solution of scientific, technical, economic tasks. For solving such problems of the real sector of the economy is often necessary to put and solve optimization problems. In turn, the optimization problem is reduced to the problem of solving a variational inequality. At the same time modern calculations are based on the application of parallel algorithms. Earlier [1, 2] were parallelized varieties extragradient methods. It was decided to consider methods of solving variational inequalities from [3] and to conduct theoretical evaluation of the efficiency of parallelization. Various possibilities were explored gradient methods, the multiplier method and extraproximal method.

In implementations of each method identified the most expensive computationally operations (for example, multiplying a matrix by a vector or matrices addition). Also stood out the blocks, the implementation of which is not dependent on the results of calculations each other.

Using the methodology of the assessment acceleration of the parallel algorithm [4] for each method was carried out the corresponding theoretical assessment. The evaluation is based on determining the share of parallel computing in the implementation of the method. In the course of work identified the most favourable from the point of view of acceleration methods. One of such methods is extragradient method for solving equilibrium problems related restrictions. We have also identified methods, the use of parallelism in which will give you less positive effect. So, for example, is a variant's extragradient method for tasks related restrictions.

In the General case, the application of parallel technologies in the implementation of methods for solving variational inequalities allows to efficiently solve important strategic tasks, for example, the task of building a sustained information systems in the Arctic regions of Russia.

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DC-APPROACH FOR EQUILIBRIUM SEARCH IN COURNOT MODEL WITH CUBIC COSTS

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We consider Cournot oligopoly model with players' S-shape costs functions [1], which are given by cubic polynomials:

$$C_i(x_i) = \alpha_i x_i^3 + \beta_i x_i^2 + \gamma_i x_i + \delta_i, \quad \alpha_i > 0, \beta_i < 0, \gamma_i > 0, \delta_i \geq 0, \beta_i^2 \leq 3\alpha_i\gamma_i, \ i \in N,$$

and with linear inverse demand function

$$p(x) = d - a\sum_{j\in N} x_j, \quad a > 0, \ d > 0.$$

Here N is a set of players and $x_i > 0$ is i-th player's output. Since such Cournot model is a potential game [2], any pure-strategy Nash equilibrium is a stationary point of the potential

$$P(x) = \sum_{i\in N}\left[-\alpha_i x_i^3 - (a+\beta_i)x_i^2 + \left(d - \gamma_i - \frac{a}{2}\sum_{j\neq i} x_j\right)x_i\right],$$

which is obviously nonconcave function. Global maximum of P is always equilibrium point.

In order to find global maximum we suggest to use branch-and-bound method, where upper bound concave functions are constructed by linearization of convex term in DC-decomposition of function P.

In the report, we also present numerical experiment results.

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APPLICATIONS OF OPTIMIZATION METHODS IN ENERGY PROBLEMS

OPTIMIZATION OF THE EQUIPMENT AGE STRUCTURE IN THE INTEGRAL MODELS OF RUSSIA'S POWER SYSTEMS DEVELOPMENT [1]

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In [1] consideration is given to the optimization problem of equipment lifetime of Russia's electric power systems. The phase variable $x(t)$ is the total electrical capacity put into operation at time t. It satisfies the nonclassical Volterra equation of the first kind

$$\int\limits_{a(t)}^{t} K(t,s)x(s)ds = y(t), \quad t \in [t_0, T], \quad x(t) = x^0(t), \quad t \in [a(t_0), t_0), \tag{1}$$

where $K(t,s)$ is the efficiency factor of using at time t of the capacity unit commissioned earlier at time s; $y(t)$ is the dynamics of the available capacity given by expert; $t-a(t)$ is the lifetime of the oldest unit at time t; $x^0(t)$ is known dynamics of commissioning the capacity on $[a(t_0), t_0)$.

In [2] the modeling of the power systems development is based on the generalizing equation (1)

$$\sum_{i=1}^{n} \int\limits_{a_i(t)}^{a_{i-1}(t)} K_i(t,s)x(s)ds = y(t), \quad t \in [t_0, T], \quad a_0(t) \equiv t, \quad a_n(t_0) \le t_0, \tag{2}$$

in which the i-th term corresponds to the i-th age group of the system elements, and properties of functions $a_i(t)$, $i = \overline{1,n}$, depend on the hypotheses assumed about the mechanism of the equipment aging.

In [1] the functional of total costs to commission new and repair and maintain the existing equipment for the $T - t_0$ (years) is used as an optimization criterion of selection of $a(t)$.

In development [1], [2], this paper considers the formulation of the optimal control problem with the same functional and integral constraint on the phase variable in the form (2), and can be optimizable functional parameters both a_i and K_i. Now we are developing a heuristic algorithm optimization of function $a_n(t)$, which determines the dynamics of removing for the older age group elements from service.

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MODELLING OF THE UNIFIED NATIONAL ELECTRICAL NETWORK DEVELOPMENT

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The unbundling of the electricity business has raised new challenges that the restructured industry faces. Expanding the transmission network through the construction of new lines or the update of existing ones increases the amount of electric energy that can be traded and the number of generators and consumers that can take part in the market. Transmission system provides nondiscriminatory, effective and competitive environment for participants. Hence, the traditional cost-minimization formulation is no longer suitable to solve transmission expansion problem in market conditions. To evaluate efficiency of network expansion the comparison of investment and social welfare is needed [1]. This requires a coordinated approach to the optimization of the generation and transmission operation and development. The mathematical model considered in the paper is a mixed-integer linear programming formulation for the static transmission expansion problem in the conditions of electric energy market. The model allows to take into account discrete nature of investment, different operating conditions while modeling market with perfect competition. The proposed methodology is applied to the network of unified energy system of Russia based on the real data. The obtained results are verified and compared with official forecasts.

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ON SEARCH OF EQUILIBRIUM IN ORIRES MARKET MODEL

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This report is dedicated to the equilibrium search problem of ORIRES market model, which was considered in [1]. In general case a behavior of company $l \in L$ could be written down as the following linear programming problem:

$$\sum_{i\in I}\sum_{s\in S}\sum_{t\in T}\tau_s^w(p-c_{li})x_{list}+\sum_{i\in I}\sum_{s\in S}\sum_{t\in T}\tau_s^h(p-c_{li})y_{list}-f\sum_{i\in I}k_{li}(z_{li}-z_{li}^0)-$$
$$-\sum_{i\in I}b_{li}^0 z_{li}\to max, \tag{1}$$

$$\alpha_{lis}z_{li}\le x_{list}\le\beta_{lis}z_{li},\quad i\in I,\quad s\in S,\quad t\in T, \tag{2}$$

$$\alpha_{lis}z_{li}\le y_{list}\le\beta_{lis}z_{li},\quad i\in I,\quad s\in S,\quad t\in T, \tag{3}$$

$$z_{li}^0\le z_{li}\le\bar{z_{li}},\quad i\in I, \tag{4}$$

$$\tau_s^w\sum_{t\in T}x_{list}+\tau_s^h\sum_{t\in T}y_{list}\le g_s z_{li},\quad i="HPS",\quad s\in S, \tag{5}$$

$$\sum_{t\in T}x_{list}\le\eta d z_{li},\quad i="PSPP",\quad s\in S, \tag{6}$$

$$\sum_{t\in T}y_{list}\le\eta d z_{li},\quad i="PSPP",\quad s\in S, \tag{7}$$

where x_{list} and y_{list} — working capacity of i type station, belonging to the company l, in season s in hour t in working days and holidays correspondingly; z_{li}, z_{li}^0 and $\bar{z}_{li}$ — installed, minimum and maximum allowed capacity of i type station, belonging to the company l, correspondingly; p — price per unit of electric power; c_{li} and b_{li}^0 — unit costs for electricity generation and unit operating costs of i type station, belonging to the company l, correspondingly; α_{lis} and β_{lis} — minimum and maximum allowed capacity coefficient of i type station, belonging to the company l; τ_s^w and τ_s^h — number of working days and holidays in season s correspondingly; g_s и d — maximum number of hours of HPS (Hydro Power Plants) used in season s and PSPP (Pumped Storage Power Plants) installed capacity per day correspondingly; η — coefficient of efficiency of PSPP.

However, behavior of some companies is specific. There are companies, which don't plan to develop power in the years immediately ahead, and, by virtue of their geographical location, haven't HPS and PSPP. When modeling behavior of such companies, only working power constraints will be presented (2)-(3), using which one can develop a solution analytically. Use of this approach decrease computational complexity of the equilibrium search problem in the ORIRES market model, which is important in view of high dimensionality of the problem under consideration.

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EXTREME SYSTEM OF CONTROLLING THE EMITTANCE INTENSITY OF THE WELL HYDRODYNAMIC GENERATOR

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At present for intensification of oil influx in the bottom-hole zone of stratum the wave generators of types working in the wide range of frequencies are effectively used [1].

During maintenance of emitter in the well conditions mismatching of rods vibration with pressure fluctuations in the jet can take place, which reduces emittance intensity.

Supposing that the own frequency of rods vibrations almost does not change, the principle opportunity of developing automatic extreme controlling system regulating pump flow rate in the mouth of the hole appears. This system will be able to secure resonance regime of rods vibrations of emitter by information about maximum of pressure drop amplitude.

By considering the well as hydraulic line with distributed parameters and choosing extreme discrete regulator of step type [2], the system modeling was done.

The results of numerical analyses showed that by finishing transition process autovibrations appear near the point of extremum, which confirms the opportunity of developing extreme system of controlling the emittance intensity by the rod hydrodynamic generator, located in the well at the oil stratum level.

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PROBLEM OF DISCRETE-CONTINUOUS OPTIMIZATION OF HEAT NETWORKS AND POSSIBLE APPROACHES TO THEIR SOLUTION

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The report considers the problem of optimization of hydraulic modes of heat supply systems (HSS) based on economic criteria for the structure running pumping equipment.
Solving this problem inhibited by factors of: HSS high dimensionality, nonlinearity flow distribution model, discrete search area on the structure of equipment, the simultaneous presence of continuous control methods, etc. So there are currently no workable methods or software systems that are suitable for practical use.
The report examines the issues of adaptation to existing HSS in ESI SB RAS basis in calculation methods acceptable and optimal modes of pipeline systems [1]. Consider a new formulation of the problem consists in determining the number of pumps running at the pumping station. It meets the requirements of uniformity of the pump on HSS in their parallel operation and gives a considerable gain in computational complexity.
To select the optimal composition of pumps connected proposed four methods: exhaustive search method, discrete branch and bound method, branch pruning method, continuous branch and bound method. These methods assume directed search options. In the optimization of each option for continuous variables is performed by internal points [2, 3].
These methods are implemented in the form of research programs and tested on a model of HSS. Computational experiments demonstrate the effectiveness of continuous branch and bound method.

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NEW OPTIMIZATION ALGORITHM FOR CONTROLLING CASCADING FAILURES IN POWER SYSTEMS[1]

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Power systems are in the core of the infrastructure of modern society. The reliability of a large number of life-support systems depends on the entire power systems reliability and it components functioning. However, the complexity of such networks makes it difficult to manage them, and parts of the network might fail. E.g. the blackout that occurred on August 14th, 2003 in the US and Canada due to both a lack of coordinated countermeasures and too long a delay between the initial emergency and the countermeasures taken, transmission lines started failing, causing a cascade of failures. The main means of counteracting the development of such severe outages in Electrical Power Systems (EPS) are so-called Emergency Control Systems (ECS). In general, ECS can be divided into centralized and local systems. A centralized ECS is a complex information system, whose main task is to provide control actions for a number of different types of disturbances offline. Typically, centralized ECS collects information on the current state of the EPS, and based on a detailed analysis of the current situation calculates coordinated control actions for a large set of possible disturbances. However, their reliability depends on many communication channels. Moreover, the rate of decision making strongly depends on the transmission speed of the slowest information channel. Due to delays in computation and the dynamical nature of the power system, the returned control actions might not be valid anymore. Finally, different power companies might put restrictions on the centralization and accumulation of information due to confidentiality requirements. Local ECS are devices that implement control actions on the basis of local information of the current state. Although their decision times are quite fast, the uncoordinated nature of their actions can create a catastrophic deterioration of the emergency due to cascading failures. In particular, recent studies have shown that the main reason for most of the recent blackouts was a cascade tripping of transmission lines due to overload [1]. Thus, there is a need to supplement existing control methods with new one that can react better and faster to changes than centralized ECS, while still being able to provide a better coordinated solution than existing local ECS can provide. The paper proposes a new optimization algorithm for load shedding to eliminate overloads in electric power systems. Lets outline the main features of proposed algorithm. 1) An optimal balance between complexity and simplicity: it is generic enough to provide high level of control actions precision and it is simple enough to meet strict time restrictions. 2) It meets high level of reliability: it does not fail in the case of the uncritical loss of information. High speed of self-healing monitoring is provided by the maximum simplification of the algorithm. This is a joint work with O. Khamisov, D. Sidorov, N. Tomin and A. Osak.

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UNCERTAINTY MANAGEMENT IN HYDROPOWER OPTIMIZATION PROBLEMS

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Hydropower problem characterized by high levels of uncertainty, which is evident at all stages of the information process in the decision management. Therefore, the search for methods and approaches to building effective decisions under uncertainty is an important and practically significant problem. Various hydrological problems with stochastic data studied in stochastic hydrology and hydrological optimization problems are considered in the stochastic programming. Occupy a special place optimization problems with uncertain input data. In a case where input parameters comprise uncertainty used Uncertain Programming. Uncertain programming is a theoretical basis for solving optimization problems under different types of uncertainty.

Currently, many problems under uncertainty are using the concept of uncertainty management, which increasingly appears in russian and foreign publications. The meaning of this concept has not yet been established and is often used along with concepts such as risk management, information uncertainty and so on. Our interpretation of the notion of uncertainty management is primarily concerned with the development of numerical methods and procedures that can help reduce the level of uncertainty depending on the type, nature, specific characteristics, volume and sources of information at all stages of the process, the accompanying management decisions. In an article for solving optimization hydropower problems with random input data is proposed to use the random programming [1,2], which is a section of mathematical programming and numerical probabilistic analysis to construct the probability density function to describe the set of possible optimal solutions for linear and nonlinear optimization problems. Under this approach, implemented procedure of propagation of uncertainty in order to reduce the level of uncertainty. At the end we consider the example of solving the optimization hydroelectric power generation problem, which depends on the prognosis of lateral inflow into the reservoir as a stochastic function.

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ONE PROBLEM OF POWER EFFICIENCY OF METHODS OF PRODUCING NITROGEN AT OIL EXTRACTION SITES

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In the given report the sphere of nitrogen usage and three main methods of its producing by air-separating units: membranous, adsorptional and cryogenic are considered . Also, actuality of nitrogen production increase and decrease of power expenditures by its receiving are shown by the author.

Having compared three methods of nitrogen production, the author made calculating - theoretical research with the aim of defining power expenditures for each method provided equal capacity of air-separating unit, but with different purity of the received nitrogen. Analysis of the received results showed, that the most effective method is adsorptional and in some cases - membranous.

The received results can be useful by the choice of the type of air-separating unit for various technological processes in industry.

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POWER SYSTEMS PARAMETERS FORECASTING USING THE HILBERT-HUANG TRANSFORM AND MACHINE LEARNING[1]

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A novel hybrid data-driven mathematical forecasting model is developed in order to increase the efficiency of short-term forecasting of non-stationary time-series. Suggested approach combines the effective tool of non-stationary time series analysis based the Hilbert-Huang integral transform (HHT) and machine learning methods. The original time series is decomposed by empirical modal functions that are then subject to the Hilbert transform in order to compute instantaneous amplitudes and frequencies at every time moment. Then the resulting modal functions and instantaneous amplitudes are used to automatically find optimal combinations of input variables for further application of machine learning models. Models that are examined include neural networks, support vector machines, the regression trees approach: random forest and boosting trees.

The random forests and gradient boosting trees learning techniques were examined. The decision tree techniques were used to rank the importance of variables employed in the forecasting models. The Mean Decrease Gini index is employed as an impurity function. The resulting hybrid forecasting models employ the radial basis function neural network and support vector regression.

Proposed strategy was tested on wind speed forecasting problem using real data acquired from the Atlantic offshore buoy data.

This is a joint work with Paul Leahy (Sustainable Energy Research Group, University College Cork, Cork, Ireland), Nikita Tomin, Victor Kurbatsky, Vadim Spiryaev (Dept. Electric Power Systems, Melentiev Energy Systems Institute, Irkutsk, Russia) and Denis Sidorov (Dept. Electric Power Systems, Melentiev Energy Systems Institute, Irkutsk State University Irkutsk, Russia)

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