ON NUMERICAL METHOD FOR GLOBAL SEARCH IN NONCONVEX OPTIMAL CONTROL PROBLEMS[1]

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Consider the following linear control system:

$$\dot{x}(t) = A(t)x(t) + b(u(t),t) \qquad \overset{\circ}{\forall}\, t \in T = [t_0, t_1], \quad x(t_0) = x_0, \tag{1}$$

$$u(\cdot) \in \mathcal{U} = \{\, u(\cdot) \in L^r_\infty(T) \mid u(t) \in U \qquad \overset{\circ}{\forall}\, t \in T \,\}, \tag{2}$$

where $A(\cdot)$ is $(n \times n)$-matrix function with continuous elements $t \mapsto a_{ij}(t)$, $i, j = 1, 2, \ldots, n$ on $T := [t_0, t_1]$, and U is a compact set. Assume also that vector function $(u,t) \mapsto b(u,t)$ is continuous with respect to variables $u \in I\!R^r$, and $t \in T$.

We study the following optimal control (OC) problem:

$$(\mathcal{P}): \qquad J(u) = F_1(x(t_1,u)) + \int\limits_T F(x(t,u),t)\, dt \downarrow \min_u, \quad u(\cdot) \in \mathcal{U}. \tag{3}$$

Functions $x \mapsto F_1(x)\colon I\!R^n \to I\!R$ and $(x,t) \mapsto F(x,t)$ (see (3)) are d.c. functions (A.D. Alexandrov's functions, see [1]), therefore these function can be represented as a difference of two convex functions with respect to variable x (for all $t \in T$):

$$F_1(x) = g_1(x) - h_1(x), \qquad F(x,t) = g(x,t) - h(x,t) \quad \forall x \in \Omega \subset I\!R^n, \quad t \in T, \tag{4}$$

where $x \mapsto g_1(x)$, $x \mapsto h_1(x)$, $x \mapsto g(x,t)$, and $x \mapsto h(x,t)$ are convex functions with respect to variable x for all $t \in T$.

Note that the OC problem $(\mathcal{P})$ is nonconvex (см. [1,2]), and it might possess a number processes, satisfying the Pontryagin maximum principle (PMP), which are rather far from a global solution. This nonconvexity is created by objective functional $J(\cdot)$ of problem $(\mathcal{P})$.

On the basis of the global optimality conditions [1] we propose the method for searching globally optimal processes in problem $(\mathcal{P})$, which combines a local search search procedure and the procedure for improving process, satisfying PMP [2]. Under certain assumptions we prove the theorem of convergence of the global search method. Also we performed a numerical experiment, which showed operability and efficiency of the developed method on a series of test problems with quadratic nonconvex objective functional.

REFERENCES

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2. A.S. Strekalovsky, M.V. Yanulevich. *Global Search in a Nonconvex Optimal Control Problem.* — Journal of Computer and Systems Sciences International. — 2013, Vol. 52, № 6, pp. 893–908.

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