Unified Modeling and Theory for Global Optimization

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ABSTRACT:

Traditionally, global optimization problems in literature are usually formulated as

$$\min f(x) \quad \text{s.t.} \quad g(x) \leqslant 0$$

where the “objective” function $f(x)$ and constraint $g(x)$ are assumed to be differentiable or simply Lipschitzian. It is well-known in nonlinear analysis [1] and mathematical physics [2] that a real-valued function is called *objective* only if it satisfies the *frame-invariance principle* [3]. However, in mathematical programming the objective function has been misused with other concepts such as cost, target, utility, and energy functions, etc [4]. Clearly, without detailed structural information on $f(x)$ and $g(x)$, it is difficult (or impossible) to have a general theory and effective methods for solving this artificially challenging problem. This could be one of main reasons why there has been no fundamental break-through in nonconvex programming over the past 50 years.
In this plenary lecture, the speaker will first present a unified mathematical model:

$$\min P(x) = W(Dx) + F(x) \quad \text{s.t.} \quad x \in X$$

where D is a linear operator, $W(y)$ is an objective function, i.e. $W(y) = W(Qy) \quad \forall\, Q^T = Q^{-1}$, which depends only physical property of the system; while $F(x)$ is a “subjective” (or cost) function which depends each problem and must be linear. The feasible set X contains only linear constraints (boundary conditions). The speaker will show how the canonical duality-triality theory [2] is naturally developed, why this theory can be used not only for model complex systems within a unified framework, but also for solving a large class of nonconvex, nonsmooth, and discrete problems in both nonlinear analysis and global optimization. Some fundamental and conceptual mistakes in recent papers by C. Zalinescu and his co-workers will be revealed. Applications will be illustrated by a list of global optimal solutions to some well-known challenging problems in global optimization, nonlinear PDEs, and information technology [5,6], and in certain cases, the global minimal solution could be the worst decision.

This talk should bring some fundamentally new insights into complex systems theory, nonlinear optimization and computational science.

References:
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