VARIATIONAL OPTIMALITY CONDITIONS WITH FEEDBACK DESCENT CONTROLS THAT STRENGTHEN THE MAXIMUM PRINCIPLE

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The talk is devoted to necessary optimality conditions for the optimal control problem (P):

$$\dot{x} = f(t,x,u), \quad x(t_0) = x_0, \quad u(t) \in U, \quad t \in T = [t_0, t_1], \tag{1}$$

$$J[x,u] = l(x(t_1)) \to \min .$$

Here, U is a given compact set. The function f is assumed to be continuous, locally Lipschitzian in x, and such that (s.t.) the set of all admissible trajectories of the control system is relatively compact in $C(T, R^n)$. The cost function l is locally Lipschitz continuous (for simplicity, below we assume that l belongs to the class $C^2(R^n)$).

Though problem (P) is stated within the conventional class $\mathcal{U} = L_\infty(T,U)$ of open-loop controls, the discussed optimality conditions are formulated in terms of feedback controls, which are arbitrary single-valued functions $v : T \times R^n \to U$. As concepts of a solution to (1) under a feedback control v we employ both Caratheodory solution concept, and Krasovskii-Subbotin constructive motions. Let $\mathcal{X}(v)$ denote the union of all solutions of these types. A control v is said to be a *descent control* (with respect to the functional J) at an admissible point $\bar{\sigma} = (\bar{x}, \bar{u})$ if there exists $x \in \mathcal{X}(v)$ s.t. $l(x(t_1)) < J[\bar{\sigma}]$. Clearly, optimality of $\bar{\sigma}$ in (P) implies the absence of descent controls at $\bar{\sigma}$.

In the considered optimality conditions, feedback controls, which are expected to be descent at a reference point $\bar{\sigma}$, are designed by means of an extremal aiming with an arbitrary support majorant of the functional J at $\bar{\sigma}$ (i.e., with a weakly decreasing solution $\varphi(t,x)$ of the respective Hamilton-Jacobi inequality with a certain boundary condition). In the simplest case, support majorants are defined by solutions of the adjoint system, while the respective optimality conditions provide a straightforward strengthening of the Maximum Principle. Below, we formulate such a strengthening for a non-smooth problem (P).

Let $\Psi(\bar{\sigma})$ denote the set of all solutions to the Clarke adjoint inclusion:

$$-\dot{\psi}(t) \in \partial_x[\psi(t) \cdot f(t, \bar{x}(t), \bar{u}(t))], \quad \psi(t_1) = l_x(\bar{x}(t_1)),$$

$$p(t,x) := \psi(t) + l_x(x) - l_x(\bar{x}(t)), \quad \psi \in \Psi(\bar{\sigma}),$$

$$U_\psi(t,x) := \operatorname*{Argmin}_{u \in U} p(t,x) \cdot f(t,x,u),$$

and let $\mathcal{V}_\psi$ denote the set of all selections of the multifunction $U_\psi(t,x)$.

Theorem (Feedback Minimum Principle). *Assume that a process $\bar{\sigma} = (\bar{x}, \bar{u})$ is optimal in problem (P). Then there exists $\psi \in \Psi(\bar{\sigma})$ s.t. the trajectory $\bar{x}$ is optimal in the following extremal problem*:

$$l(x(t_1)) \to \min; \quad x \in \bigcup_{v \in \mathcal{V}_\psi} \mathcal{X}(v).$$

As one can easily observe, this theorem encloses the non-smooth Maximum Principle by F. Clarke for the addressed optimal control problem. Note that, the proposed optimality condition is of a variational type, since it is formulated in terms of an auxiliary infinite-dimensional extremal problem. Moreover, further generalizations of these conditions with support majorants are of variation types as well. In the modern Hamilton-Jacobi Theory, these results are the first necessary conditions, which are comparable in efficiency with the Maximum Principle.