OPTIMAL CONTROL OF TRAJECTORY BUNDLES

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We consider an optimal control problem of the form

$$\frac{dx}{dt} = f(t,x,u), \quad t \in T_0 = [0,T], \tag{1}$$

$$\frac{\partial \rho}{\partial t} + \left\langle \frac{\partial \rho}{\partial x}, f(t,x,u) \right\rangle = -\rho \operatorname{div}_x f(t,x,u), \quad t \in (0,T),\ x \in \Omega, \tag{2}$$

$$x(0) \in M_0, \tag{3}$$

$$\rho(0,x) = \rho_0(x), \quad x \in \overline{\Omega}, \tag{4}$$

$$u(t) \in U,\ t \in T_0, \tag{5}$$

$$I(u) = \int_{T_0} \int_{M_{t,u}} F(t,x,\rho(t,x))\, dxdt + \int_{M_{T,u}} \varphi(x,\rho(T,x))\, dx \to \inf. \tag{6}$$

Here, $x : T_0 \to \mathbf{R}^n$, $\rho(t,x) : T_0 \times \overline{\Omega} \to \mathbf{R}^1$, $u : T_0 \to \mathbf{R}^r$, Ω is a connected open subset of $\mathbf{R}^n$, the set M_0 is compact in $\mathbf{R}^n$, $M_0 \subset \Omega$, and "overline" stands for the closure of a set.

The standing assumptions are as follows:
1) U is a compact subset of $\mathbf{R}^r$.
2) $f(t,x,u)$ is continuous and continuously differentiable in all variables, and satisfies Lipschitz and linear growth conditions with respect to x.
3) $F(t,x,\rho)$, $\varphi(x,\rho)$, and $\rho_0(x)$ are nonnegative, continuously differentiable functions.

By admissible controls we mean smooth functions $u(\cdot)$ with values in U, i.e., $u(\cdot) \in C^1(T_0, \mathbf{R}^r)$, $u(t) \in U$, $t \in T_0$. Given an admissible control function $u(\cdot)$, consider a bundle $\cup_{x_0 \in M_0} x(\cdot, x_0; u)$ of solutions to equation (1), starting from the set M_0. The set $M_{t,u}$ is defined as the section of the bundle at a given t, i.e.,

$$M_{t,u} = \{x_t = x(t,x_0;u) : x(\cdot,x_0;u) \text{ is a solution to (1), (3) under control } u(\cdot),\ x_0 \in M_0\}.$$

Problem (1)–(6) consists in optimal control of trajectory bundles considering the density of distribution. Previously, in [2], modeling issues and analysis of dynamics of trajectory bundles were addressed, and necessary conditions for optimality were given. Note that, in a number of practical applications, controls are smooth. In this work, by using technique [1], we propose necessary conditions for optimality in the class of smooth control functions.

REFERENCES

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2. D.A. Ovsyannikov. *Mathematical methods for optimization of dynamics of bundles.* Lecture Notes. Leningrad, Leningrad State University, 1986, 92 p.