Game Mathematical Model to Study Power System Interconnections in View of Economic Interests of Participants: Preliminary Consideration

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Abstract – Simplified game-ORIRES model is developed. “Russian Far East – Republic of Korea” potential power system interconnection is studied by means of the model. To analyze the model Stakkelberg method is applied. Preliminary results obtained by the model, in particular compromise prices for electricity traded between players and mutually beneficial for them, are discussed. Ways of model further development are discussed.

Index terms – Mathematical model, interstate electric tie, price, cost, power system interconnection.

I. INTRODUCTION

Game ORIRES model is being developed by Energy Systems Institute (ESI) to study cost and benefit of power system interconnection (PSI) from the viewpoint of each participant/player engaged into PSI. The mathematical model is based on game theory which is supposed to be the most appropriate tool for analysis of electric power system (EPS) potential interconnection and construction of interstate electric ties (ISETs) in market environment.

Game mathematical model stems from liner ORIRES model 0. The latter computes optimal transfer capabilities of ISETs, mix of generating capacities and operating conditions (in terms of merit order loading of power plants – PPs) of EPSs and power exchange via ISETs. Liner ORIRES model optimizes parameters of PSI as if it were a single entity. However potential power interconnections will comprise various EPSs of different countries having their own economic interests. These interests have to be taken into account while modeling PSI.

II. SET UP OF THE PROBLEM

To take into consideration economic interest of each player, incorporated into a power system interconnection, it should be represented in game mathematical model by its own separate objective function. Such a function is represented by expression (1). This is function of annuitized cost. It was thoroughly described in 0, which readers are referred to.

Each country/player acts to minimize its objective function (1) in view of constraints (and objective functions of other players). Among constraints are balance equations of installed and operating capacities, constraints on development of PPs and electric ties, operating capacity of PPs, power flows via transmission lines, total annual electricity generation by power plants with limited output (hydraulic and pumped storage PPs). The aforementioned equations and constraints are found in 0 and not principally changed while modifying linear ORIRES model (1; see the next page).

III. SIMPLIFIED GAME MODEL

For the beginning the simplified version of the model was considered. Taken into account were two nodes representing Russian Far East (RFE) and Republic of Korea (ROK) EPSs (these nodes_EPSs in fact represent different players operating on international power market), one type of PP for each node and one hour (time of maximum load) for interconnected EPSs.
A. Stakkelberg Method

To analyze the simplified model Stakkelberg method was applied. The block diagram describing the application of the method is presented in Fig.1. Two players are considered in the method. One is assumed to be a leader, while another one being outsider. Each player has its variables to optimize. However the leader optimizes its variables first. Outsider takes the optimal variables of the leader as constraints, which can’t be changed by it and optimizes its variables in view of these constraints. The method was taken due to its relative simplicity when analytical solution of the problem can be obtained.

B. Description of the Model

The simplified objective functions for RFE and ROK with related sets of constraints are given in equations (2)-(11):

\( L_s = \sum_{j_s=1}^{J_s} \sum_{i_1=1}^{J_1} \sum_{y_1=1}^{T_1} \tau_{i_1} c_{s_j} \cdot x_{s_j y_1} + \sum_{j_1=1}^{J_1} \sum_{i_1=1}^{J_1} k_{s_j i} (r_s + b_{s_i}) X_{s_j i} + 
\)

\[ + q_s \left[ \sum_{s=1}^{S} \sum_{j_1=1}^{J_1} \sum_{j_1=1}^{J_1} \sum_{j_1=1}^{J_1} \sum_{j_1=1}^{J_1} k_{s_j s_j} (r_s + b_{s_j s_j}) X_{s_j s_j} + \sum_{s=1}^{S} \sum_{j_1=1}^{J_1} \sum_{j_1=1}^{J_1} \sum_{j_1=1}^{J_1} \sum_{j_1=1}^{J_1} k_{s_j s_j} (r_s + b_{s_j s_j}) X_{s_j s_j} \right] - 
\]

\[ - \sum_{j_1=1}^{J_1} \sum_{s=1}^{S} \sum_{y_1=1}^{T_1} \sum_{i_1=1}^{J_1} \tau_{i_1} x_{s_j y_1} + \sum_{j_1=1}^{J_1} \sum_{s=1}^{S} \sum_{j_1=1}^{J_1} \sum_{y_1=1}^{T_1} \tau_{i_1} x_{s_j y_1} \rightarrow \text{MIN}, \]

\[ \sum_{s=1}^{S} q_s = 1, \quad q_s \geq 0 \quad s = 1, S. \]

where \( K_i = k_i (r_i + b_i) \) and \( K_2 = k_2 (r_2 + b_2) \) are costs for new generating capacity in RFE/ROK, with \( k_i/k_2, r \) and \( b_1/b_2 \) being respectively specific capital investment, rate of return and fixed cost for generating capacity in the countries;

\( K = k(r + b) \) – cost for new transmitting capacity of “RFE–ROK” ISET;
\[ x_1, x_2 \] – operating capacity of RFE and ROK PPs;
\[ v_1, v_2 \] – installed generating capacity of RFE and ROK PPs;
\[ y \] – power flow via the ISET;
\[ w \] – transfer capability of the ISET;
\[ c \] – price for electricity being traded between nodes;
\[ q \] – share of cost for construction and maintenance of the ISET borne by Russia;
\[ H \] – hours of yearly utilization of generating capacity;
\[ c_1, c_2 \] – fuel cost for RFE and ROK thermal PPs;
\[ N_1^0, N_2^0 \] – installed generating capacity of existing PPs in RFE and ROK;
\[ N_1^1, N_2^1 \] – maximum possible installed generating capacity in RFE and ROK by the target year;
\[ P_1, P_2 \] – load of RFE and ROK consumers in time of maximum load for PSI;
\[ 1.2 \] – multiplier, which takes into account reserve margins.

As can be seen from the functions (2), (7) and constraints (3), (8) and (4), (9) RFE is a net exporter, while ROK is a net importer.

The input data for the model was taken for the target year 2020 from 0.

C. Discussion of the Results

First, objective functions (2) and (7) for RFE and ROK respectively were studied. It was found out that they reach their minimum magnitudes at highest transfer capability of ISET \( w \) (considering constraints (3)–(5) for RFE and (8)–(10) for ROK). The above is true under condition that price for traded electricity is higher than cost for electricity production in RFE, and lower than cost for new PP development in ROK, with cost for ISET being taken into account by countries.

The condition in fact impose additional constraint on the objective functions (2), (7) and are taken into account bellow, while finding out area of compromise prices for traded electricity.

Maximum \( w \) (\( w_{\text{max}} \)) determined from constrains (3), (5) and (8), (10) makes both objective functions under considered above condition the lowest (optimal). Then, power transfer \( y \) was assumed to be equal to \( w_{\text{max}} \). Thus, optimal power flow via ISET and optimal transfer capability of ISET as well as optimal generating capacities for RFE and ROK are determined.

After the noted transformations both objective functions become dependant on only two variables: price for traded electricity \( c \) and share of cost for ISET \( q \) borne by Russia.

Applying Stakkelberg method to both transformed objective functions two areas of prices acceptable for each participant separately were obtained. The final result of the case study was presented in Fig. 2.

Price for electricity traded between RFE and ROK as a function of share of cost for “RFE–ROK” ISET construction and maintenance borne by Russia is presented in Fig.2. The area bellow the upper line marked with squares belongs to prices for electricity imported by ROK, which are acceptable for the country. This means that cost for purchasing power from RFE (along with ROK share of cost for “RFE–ROK” ISET) is less than cost for construction and operation of new generating capacity in ROK. The line itself determines prices when there is no difference for ROK whether to expand its own generating capacity or purchase power from RFE, because costs are equal in both cases. Accordingly, prices being higher than the upper line are not acceptable for ROK.

![Fig.2. Area of price compromise for power traded between RFE and ROK.](image-url)
The area above the lower line marked with rhombuses belongs to prices for electricity exported by RFE, which are acceptable for the country. This means that cost for producing power by RFE PPs (along with RFE share of cost for “RFE–ROK” ISET) is less than benefit gained by RFE from power export to ROK. The line itself determines prices when there is no difference for RFE whether to generate power for export or not doing that, because cost is equal to benefit. Accordingly, prices being under the lower line are not acceptable for RFE.

As can be seen from Fig.2 the described above areas intersect. The intersected area is one where prices for electricity traded are acceptable both for RFE and ROK. So, this area can be called “compromise” area where compromise prices mutually beneficial for both players can be sought and found.

Finally, it is needed to note that the above case study is a testing one. The input data was taken very approximately and the model as was said earlier was very simplified. So, obtained results, in particular calculated prices for traded electricity, are very preliminary. Currently, the game model is being developed to take into account two hours (hours of maximum load in ROK and RFE), and some first tentative results are already obtained. The full–scale game ORIRES model with objective function given in (1) is expected to be developed soon.

### IV. CONCLUSIONS

1. Game ORIRES mathematical model is being developed by ESI to study cost and benefit of power system interconnection from the viewpoint of each participant/player engaged into the PSI.

2. The case study resulted in the approximate area of compromise solutions where prices for electricity traded between RFE and ROK are mutually beneficial for both players.

3. Game ORIRES model is currently being developed from simplified one to full–scale model.

### V. REFERENCES


### VI. BIOGRAPHIES

**Oleg V. Khamisov** was born in 1963. He graduated from Mathematical Department of Irkutsk State University in 1985. In 1993 he received the Doctoral degree in mathematics. The topic of his theses was: “Global Optimization of Functions with Concave Minorant on Polyhedron”. At present he is head of the laboratory of ESI. His scientific interests are: global optimization, stochastic programming, and operations research. He has nearly 40 scientific papers published.

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**Sergey V. Podkovalnikov** was born in 1958. In 1980 he graduated from Irkutsk Polytechnic Institute as an electrical engineer. In 1989 he defended the thesis on application of mathematical methods for decision-making under uncertainty and multiple criteria to energy studies and got Candidate (Dr.) of Technical Sciences Degree. In 1993 he was a participant of Young Scientists Summer Program of International Institute for Applied System Analysis (Austria). In 1997 – 1998 he was with Pace University (USA) as a Fulbright Scholar. In 2003-2004 he took part in Northeast Asia Regional Electric System Ties - NEAREST project working with Korea Electrotechnology Research Institute. He is IEEE Member. For the time being he is a Principal Researcher of Energy Systems Institute. His research interests are: methods for decision-making in energy under uncertainty and multiple criteria, interstate electric ties and interconnected power systems, expansion planning of electric power industry in market environment, electric power industry liberalization. He is the author and co-author of nearly 80 scientific papers and books.